Exercise 1

Loop Invariant

A loop invariant is a condition that is necessarily true immediately before and immediately after each iteration of a loop. A *loop invariant I* has to satisfies the following three conditions:

1. $Pre \to I$

2. $Step(I \land c, b) \to I$

3. $I \land \neg c \to Post$

where c is the looping condition, Pre is the condition to hold before entering the loop, $Post$ is the post-condition of the loop, and $Step(I, body)$ is the condition to hold after execute the function body b from the condition $I \land c$.

Coffee Can Problem

You are given a coffee can that contains beans painted either black or white. Next to the can is a large pile of black beans. One randomly draws two beans from the can. If they have different color, then return white bean to the can and throw away the black bean. If they have the same color, then throw away both beans and place a black bean from the pile into the can.

An algorithm describe the coffee can problem can be found below:

Input: $n > 0, m > 0$
$B := n, W := m$;
while $B + W > 1$ **do**
    if $B \geq 0 \land W \geq 2 \to B := B + 1, W := W - 2$ //same color
    | $B \geq 2 \land W \geq 0 \to B := B - 1$ //same color
    | $B \geq 1 \land W \geq 1 \to B := B - 1$ //different color
    fi;
end
Postcondition: will be defined later

Algorithm 1: The Coffee Can Problem

Show if each of the followings are both correct loop invariant and post condition

1. $B + W \geq 1$

2. $B + W \geq 2$

3. $B \bmod 2 = 1 \leftrightarrow n \bmod 2 = 1$