Quantifier-Free Equality and Data Structures

Lab Notes (with Z3Py)

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Uninterpreted Functions and Sorts

Function Declaration:
x = **Function**(NAME, ARG_TYPES, RET_TYPE)

Sort Declaration:
A = **DeclareSort**(NAME)

Examples:
euf1.py
euf2.py
euf3.py

Datatypes

T = **Datatype**(NAME)

Declare constructors.
T.**declare**(CONS_NAME, (ACC_NAME, TYPE)…)

# Create the datatype.
T = T.**create**()

Examples:
list1.py
list2.py
tree.py
bintree.py

Arrays

Array Declaration:
a = **Array**(NAME, INDEX_TYPE, VALUE_TYPE)

Array Access:
Select(a, INDEX)
a[INDEX]

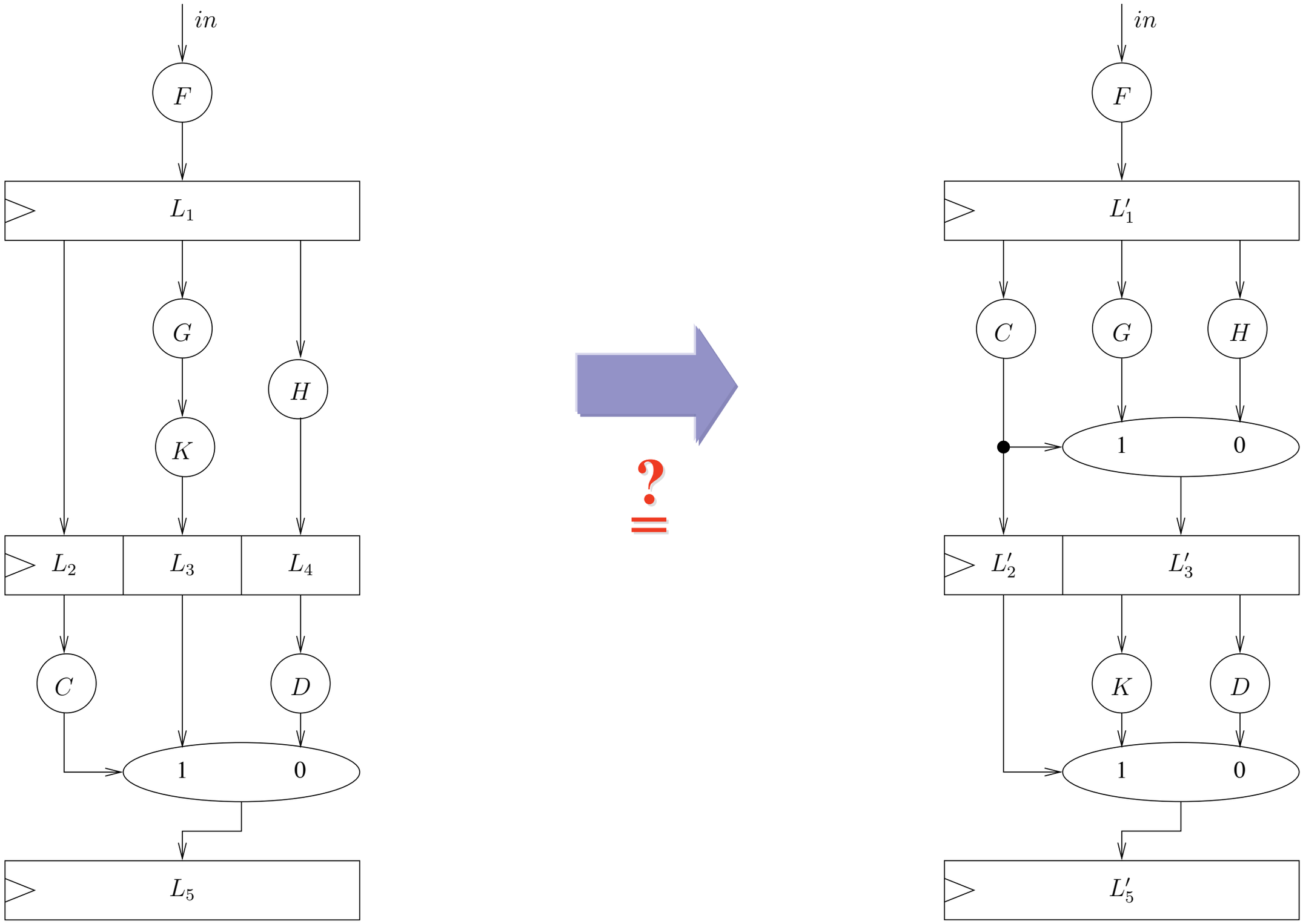
Array Update:
Store(a, INDEX, VALUE)

Examples:
array.py

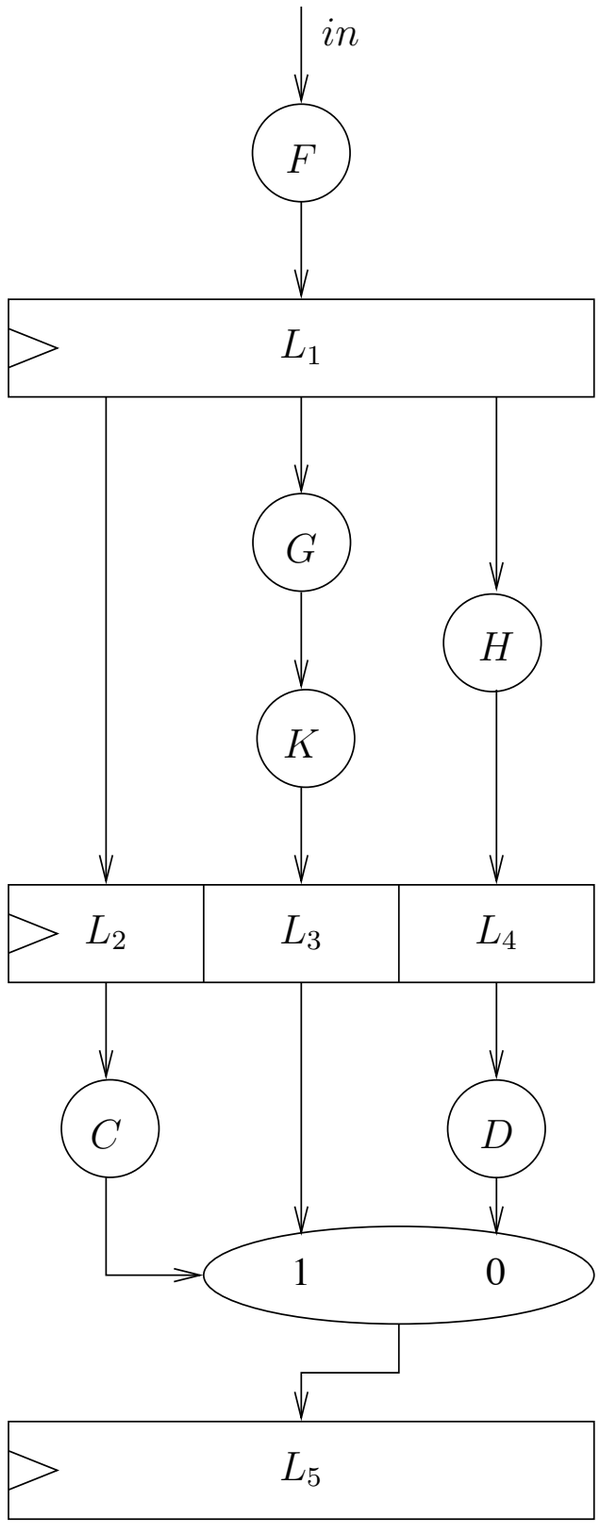
Equivalence of Circuits

- Check if two circuits are equivalent.

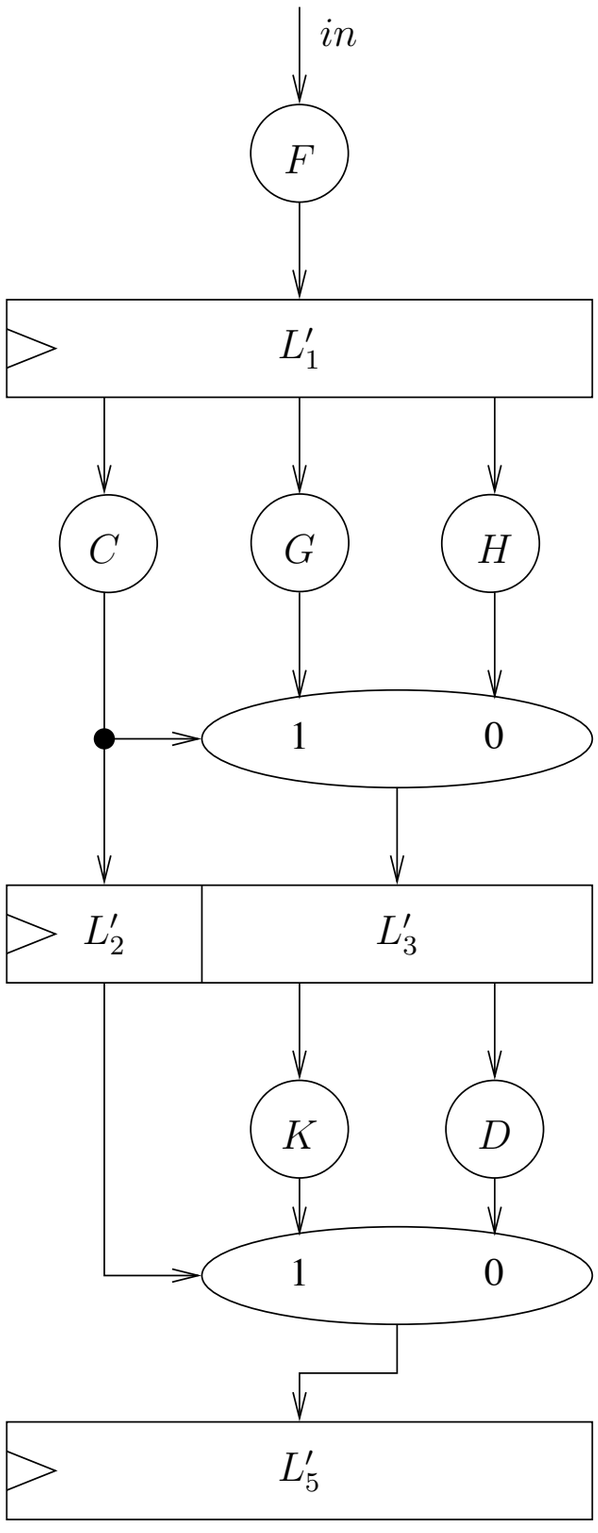
- See circuit.py.



source: http://www.decision-procedures.org/slides/uf.pdf



$$L_1 = F(in)$$
$$L_2 = L_1$$
$$L_3 = K(G(L_1))$$
$$L_4 = H(L_1)$$
$$L_5 = C(L_2)\ ?\ L_3 : D(L_4)$$



$$L_1{}' = F(in)$$

$$L_2{}' = C(L_1{}')$$

$$L_3 = C(L_1{}') \ ? \ G(L_1{}') : H(L_1{}')$$

$$L_5{}' = L_2{}' \ ? \ K(L_3{}') : D(L_3{}')$$

Exercises

- Use Z3Py to check if the following formulae are satisfiable.
  - $g(x) = h(z) \wedge x = f(g(x)) \wedge g(f(h(z))) = f(y) \wedge f(y) \neq g(x)$
  - $car(x) = car(y) \wedge cdr(x) = cdr(y) \wedge f(x) \neq f(y)$
  - $a\langle i \triangleleft x \rangle[j] = a[k] \wedge a\langle j \triangleleft y \rangle[k] = a[i] \wedge a[k] \mathrel{!}= a[j] \wedge x = y$