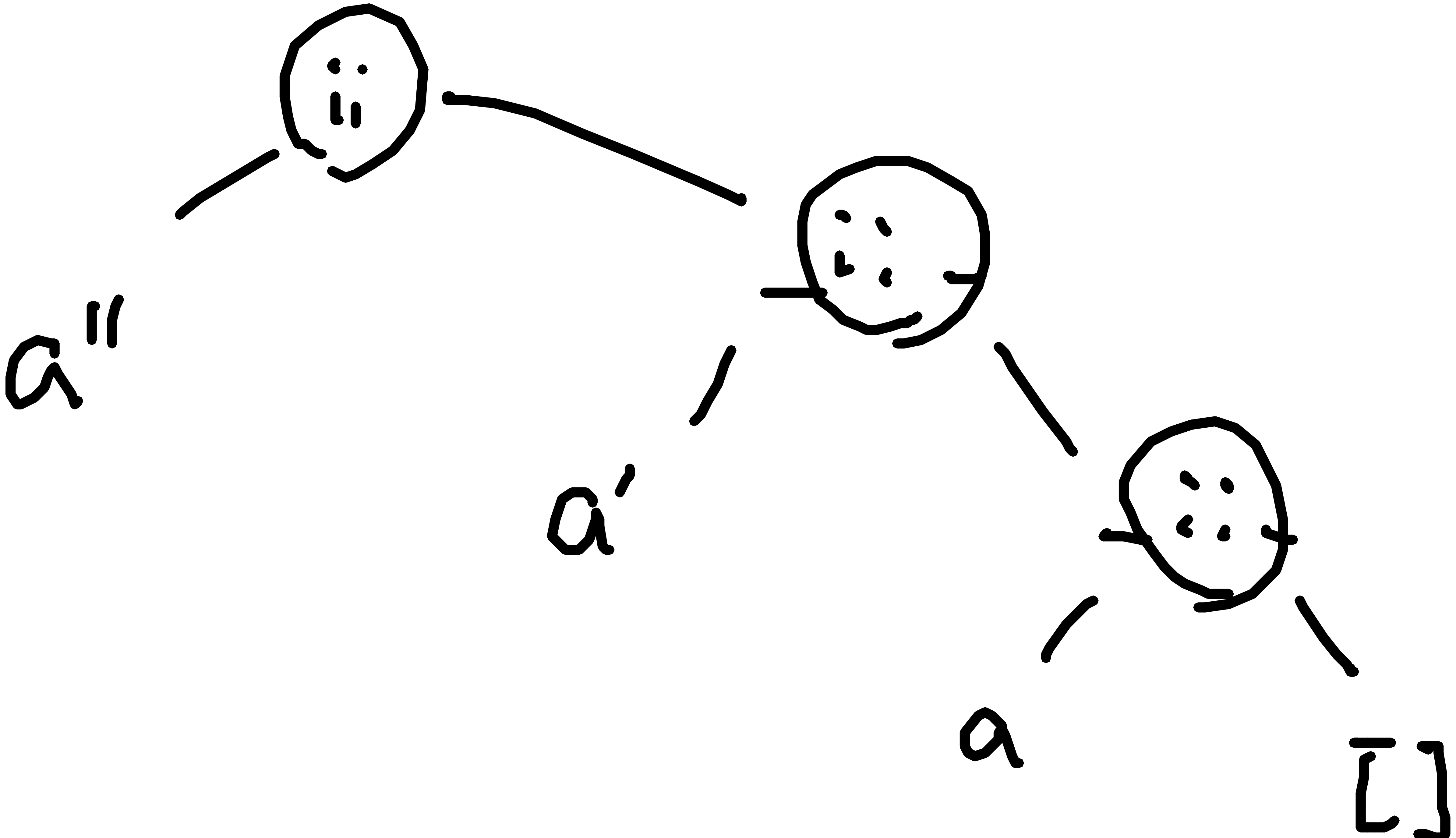
```
data List (A : Set) : Set where
  [] : List A
  _∷_ : A → List A → List A
```



λ-calculus

$\underline{\lambda x.}$ lambda abstraction

$\underline{x}$ variable

$\underline{t \ u}$ application

$(\lambda x. \lambda y. y\, x\, x)\ (\lambda z. z)$

α-conversion $\quad (\lambda w. w)$

free/bound occurrences of variables

~~free~~

$\lambda y.\ \lambda x.\ x\ y\ y$

bound

"綁"

$\perp$

$\lambda x.\ (\lambda x.\ x)\ x$

$\lambda y.\ (\lambda x.\ x)\ y$

α-equivalence:

t, u α-equi-

$t \to_\alpha^* u$

$\Pi[x:A]\ B_x$

$\Pi[y:A]\ B_y$

β - reduction

$$(\lambda x.\ t)\ u$$

$t[u/x]$: replace all free occurrences of x in t with u

free

$$(\lambda x.\ \underbrace{(\lambda y.\ x)})\ y$$

$$\not\to_\beta\ \lambda y.\ y$$

$$\to_\alpha (\lambda x.\ (\lambda z.\ x))\ y \to_\beta \lambda z.\ y$$

capture avoidance

$\lambda x . \underline{\underline{x}}$ $\qquad$ $\lambda . 0$

$\lambda y . y$

$\lambda x . \lambda y . x$

$\lambda (\lambda\ \underline{1})$

de Bruijn indices

$\lambda . (\lambda\ (x(\lambda\ 3\ )$

Church encoding

$0 := \lambda s. \lambda z. \ z$

$1 := \lambda s. \lambda z \ \ s \ z$

$2 := \lambda s. \lambda z. \ \ s \ (s \ z)$

$n := \lambda s. \lambda z \ \ \underbrace{s \ (s \cdots (s}_{n} \ z))$

$\underline{\text{plus}} \ \ \overline{m} \ \ \overline{n} \ \rightarrow^{*}_{\beta} \ \overline{m+n}$

$$(\lambda x.\ \overset{A\to B}{x}\ x)^{A\to B}\ (\lambda y.\ y\ y) \qquad A = A\to B \ ??$$

$$\|$$

$$\to_\beta\ (\lambda y.\ y\ y)\ (\lambda y.\ y\ y)$$

$$\to_\beta\ \ldots$$

$$\dfrac{\dfrac{}{A \vdash A}\ (\text{assum})}{\vdash A \to A}\ (\to I)$$

$\lambda x . x$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A\to B, A \vdash A\to B}\,(\text{assum}) \qquad \dfrac{}{- \vdash A}\,(\text{assum})}{A\to B,\ A\vdash B}\,(\to E)}{A\to B\vdash A\to B}\,(\to I)}{\vdash (A\to B)\to A\to B}\,(\to I)$$

$$\downarrow$$

$$\lambda x.\ \lambda y\ \underline{x}\ \underline{y}$$

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

$$\frac{\Gamma \vdash p \to q \qquad \Gamma \vdash p}{\Gamma \vdash q} \; (\to E)$$

$\underline{\Gamma \vDash p \to q}$

$\underline{\Gamma \vDash p}$

$\to \Gamma \vDash q$

$$V \to \cancel{\mathrm{Bool}}\ \{ S \subseteq \mathbb{R} \mid S \text{ open} \} =: \mathbb{O}$$

$$[\![\_]\!] : \mathrm{PROP} \to (V \to \mathbb{O}) \to \mathbb{O}$$

$$[\![\mathrm{atom}\ x]\!]\ \sigma = \sigma\ x$$

$$[\![p \to q]\!]\ \sigma = \overline{[\![p]\!]_\sigma}^{\circ} \cup [\![q]\!]_\sigma$$

p is true $\Leftrightarrow [\![p]\!]\sigma = \mathbb{R}$

$$[\![\bot]\!]\ \sigma = \emptyset$$

$$[\![p \vee q]\!]\ \sigma = [\![p]\!]_\sigma \cup [\![q]\!]_\sigma$$

$p \vee \neg p$ is valid

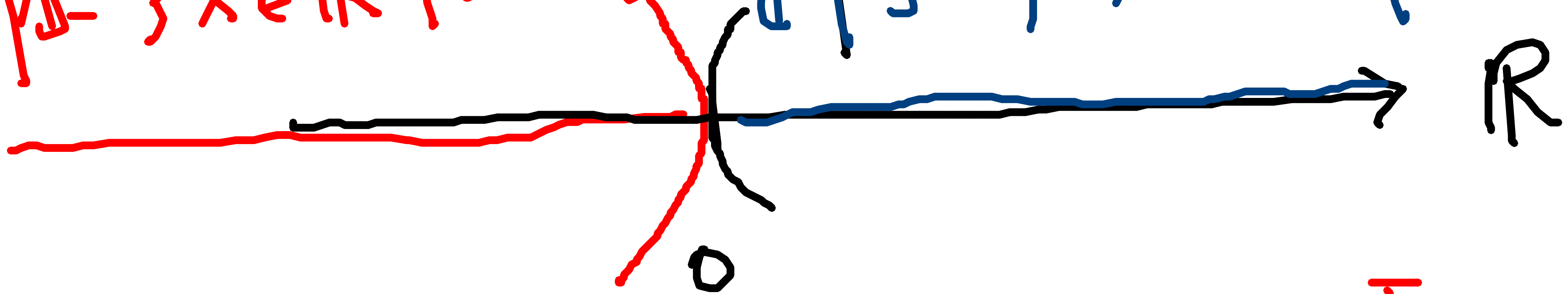
$$= \forall \sigma : V \to \mathcal{O}.\ [\![p \vee \neg p]\!]_\sigma = \mathbb{R}$$

$p \vee \neg p$ is not valid

$$\Leftarrow \exists \sigma : V \to \mathcal{O}.\ [\![p \vee \neg p]\!]_\sigma \neq \mathbb{R}$$

$[\![p]\!] = \{x \in \mathbb{R} \mid x < 0\}$

$[\![p]\!] = \{x \in \mathbb{R} \mid x > 0\}$



$$[\![\neg p]\!]_\sigma = [\![p \to \bot]\!]_\sigma$$

$$= \overline{[\![p]\!]_\sigma}^{\circ} \cup \cancel{[\![\bot]\!]_\sigma} \to \emptyset$$

$$[\![p \vee \neg p]\!]_\sigma$$

$$= \{x \in \mathbb{R} \mid x \neq 0\} \neq \mathbb{R}$$