$\text{ind}\ P\ z\ s\ (\text{suc}\ (\text{suc}\ \text{zero}))$

$= s\ 1\ (\underline{\text{ind}\ P\ z\ s\ (\text{suc}\ \text{zero})})$

$= s\ 1\ (s\ 0\ (\underline{\text{ind}\ P\ z\ s\ \text{zero}}))$

$= \underline{s\ 1}\ (\underline{s\ 0}\ z)$

suc (suc zero)

n

$\text{ind}\ P\ z\ s\ \bar{n}$

(+) 0

$s\ (n-1) \cdots (s\ 2\ (s\ 1\ (s\ 0\ z)))$

$(n-1) \cdots 2 + 1 + 0 + 0$

$\text{ind}\ (x\ \_ \to \mathbb{N})\ (+)\ 0\ n$

$\simeq \sum_{k=0}^{n-1} k$

$\mathrm{pred} : \mathbb{N} \to \mathbb{N}$

$\mathrm{pred}\ \mathrm{zero} = \boxed{\mathrm{zero}}$

$\mathrm{pred}\ (\mathrm{suc}\ \boxed{n}) = n$

$P = \lambda\_ \to \mathbb{N}$

motive

$z : P\ \mathrm{zero} = \mathbb{N}$

$z = \boxed{\mathrm{zero}}$

(pred n)

$s : \Pi[k : \mathbb{N}]\ \underbrace{P\ k}_{= \mathbb{N}} \to \underbrace{P\ (\mathrm{suc}\ k)}_{= \mathbb{N}}$

$s = \lambda n .\ \lambda\_ .\ n$

$$\mathrm{isEven} : \mathbb{N} \to \mathrm{Bool} \qquad P = \lambda\_.\ \mathrm{Bool}$$

$$\mathrm{isEven}\ \mathrm{zero} = \boxed{\mathrm{true}}$$

$$\mathrm{isEven}\ (\mathrm{suc}\ n) = \mathrm{not}\ (\mathrm{isEven}\ n)$$

$$z = \boxed{\mathrm{true}}$$

$$s = \lambda n.\ \lambda x.\ \mathrm{not}\ x$$

$$\mathrm{ind}\ P\ z\ s\ 2$$

$$= s\ 1\ (s\ 0\ z)$$

$$= \mathrm{not}\ (\mathrm{not}\ \mathrm{true}) = \mathrm{true}$$

$\mathrm{isEven} : \mathbb{N} \to \mathrm{Bool}$

$\mathrm{isEven}\ \mathrm{zero} = \boxed{\mathrm{true}}$ ← z

$\mathrm{isEven}\ (\mathrm{succ}\ n) = \boxed{\mathrm{not}\ x}$ $\quad$ $\mathrm{isEven}\ n$

x

$s = \lambda n.\ \lambda x.\ \mathrm{not}\ x$

$$\_+\_ : \mathbb{N} \to \mathbb{N} \to \mathbb{N} \qquad P = \lambda\_.\ \mathbb{N} \to \mathbb{N}$$

$$\mathrm{zero} + n = n$$

$$(\mathrm{suc}\ m) + n = \mathrm{suc}\ (m + n)$$

$$\mathrm{plus} : \mathbb{N} \to \mathbb{N} \to \mathbb{N}$$

$$\mathrm{plus}\ \mathrm{zero} = \lambda n.\ n$$

$$\mathrm{plus}\ (\mathrm{suc}\ m) = \lambda n.\ \mathrm{suc}\ (\underbrace{\boxed{\mathrm{plus}\ m}}_{f}\ n)$$

$$s = \lambda m.\ \lambda f.\ \lambda n.\ \mathrm{suc}\ (f\ n)$$

$$\_\times\_ : \mathbb{N} \to \mathbb{N} \to \mathbb{N}$$

$$\text{zero} \times n = \text{zero}$$

$$(\text{suc } m) \times n = n + (m \times n)$$

$A : \mathcal{U},\ B : \mathcal{U},\ f : A \to B,\ x : A,\ y : A$

refl

$p : \mathrm{Id}\ A\ x\ y \vdash \ :\ \mathrm{Id}\ A\ (f\ x)\ (f\ y)$

已證

欲證

$P\ x \to P\ y$

$P := \lambda z.\ \mathrm{Id}\ A\ (f\ x)\ (f\ z)$

refl (f x)

$: P\ x = \mathrm{Id}\ A\ (f\ x)\ (f\ x)$

$P\ y = \mathrm{Id}\ A\ (f\ x)\ (f\ y)$