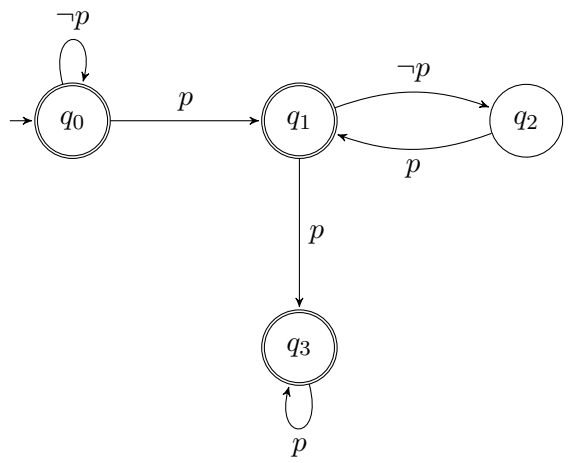
Suggested Solutions to Homework Assignment

[Compiled on July 7, 2011]

1. (20 points) Define a Büchi automaton (by drawing its state diagram) for each of the following temporal properties.

   (a) Once p holds, it should hold at every second next position.

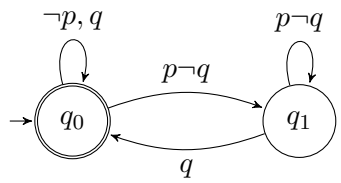
   Solution.



   □

   (b) Whenever p holds, q will hold eventually at a strictly later position and before then p continues to hold.

   Solution.



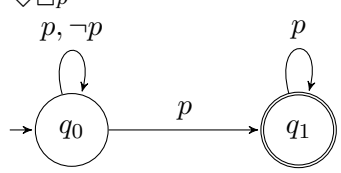
   □

2. (20 points) Consider defining a Büchi automaton for the intersection of two temporal properties $\Diamond\Box p$ and $\Box\Diamond q$.

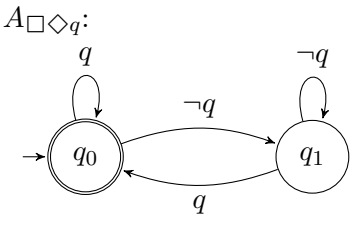
   (a) Apply the intersection (synchronous product) construction discussed in class to define the automaton; please draw the state diagram.

   Solution.

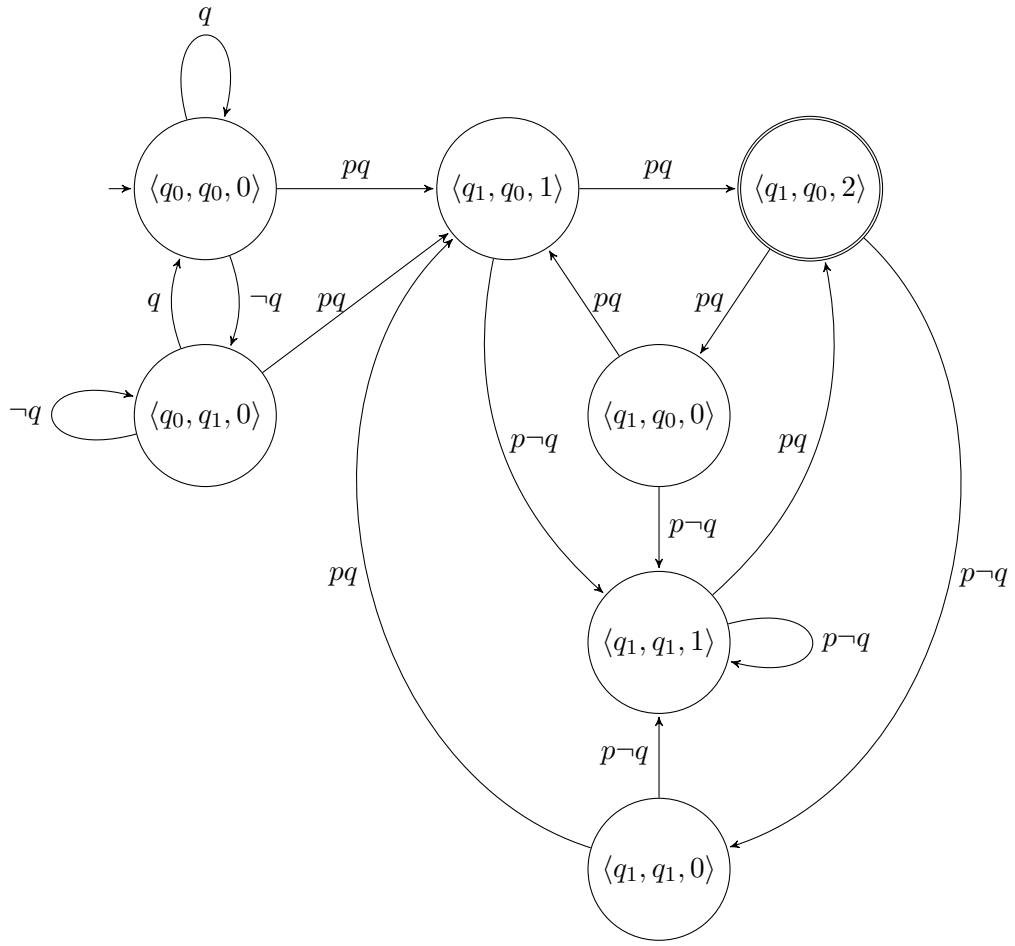
   $A_{\Diamond\Box p}$:



   $A_{\Box\Diamond q}$:



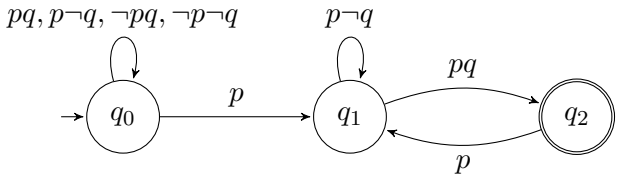
Intersection of $A_{\Diamond\Box p}$ and $A_{\Box\Diamond q}$:



□

(b) Try to find another Büchi automaton that is smaller and yet (language) equivalent to the preceding automaton.

Solution.

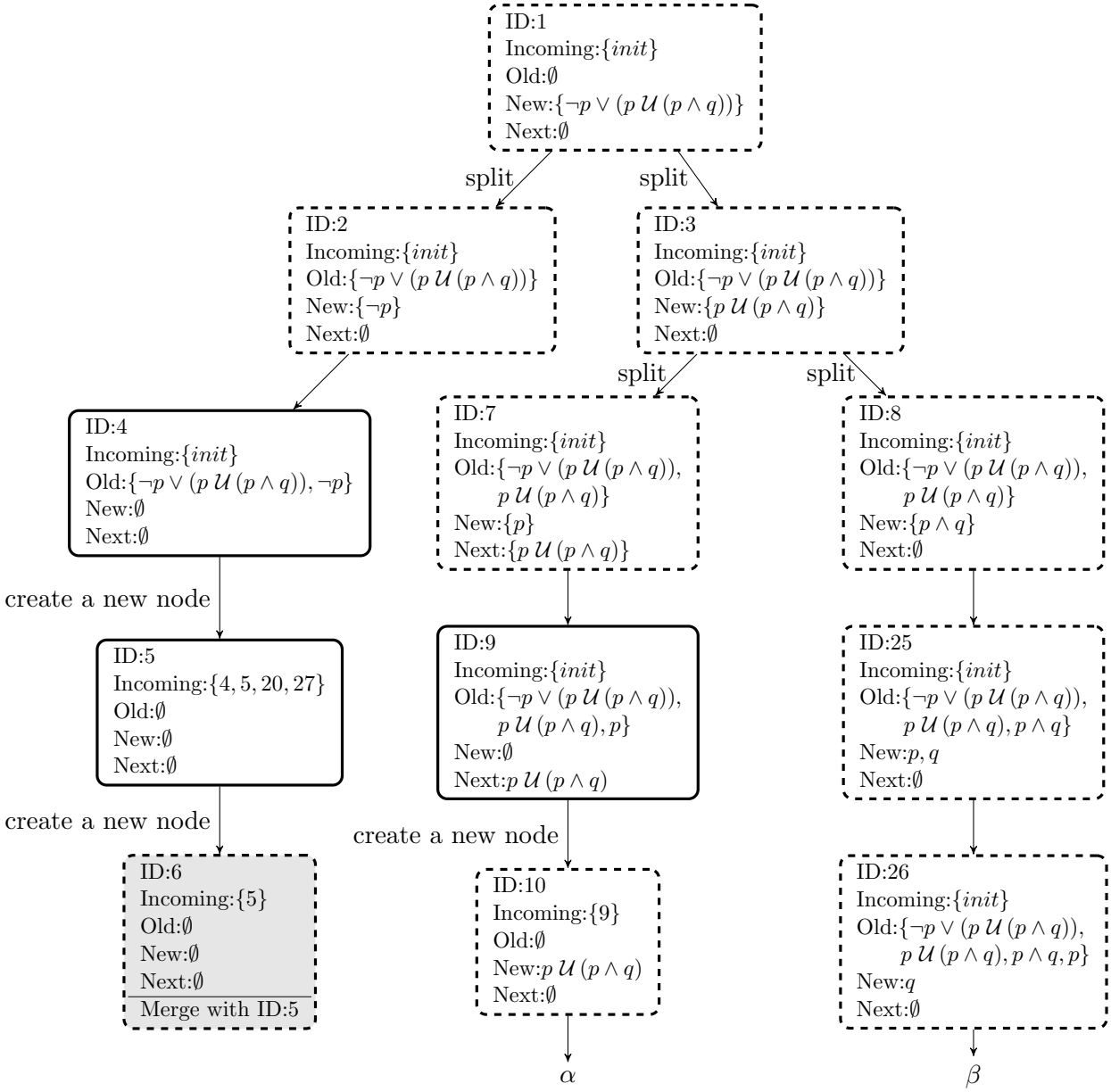


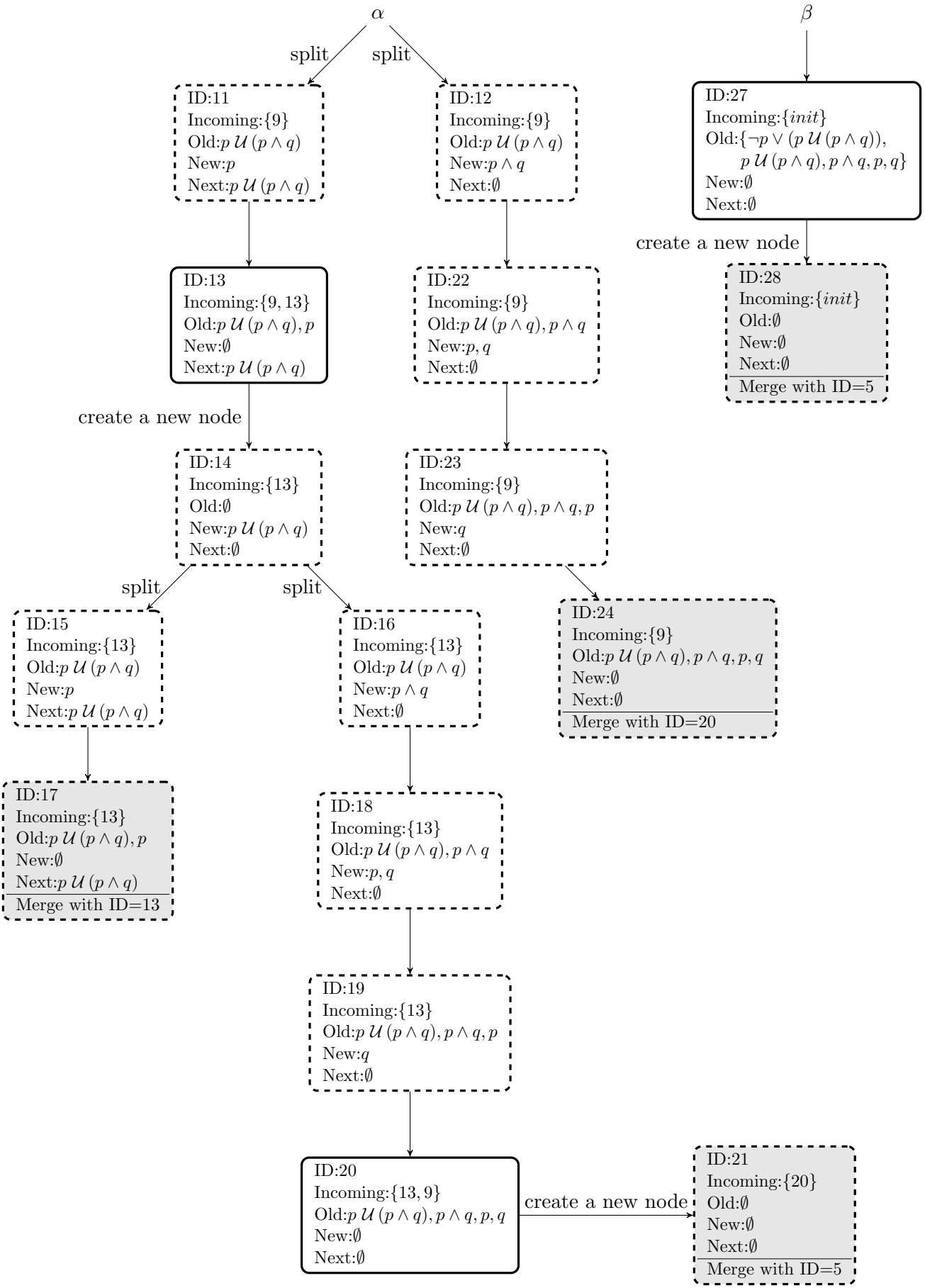
□

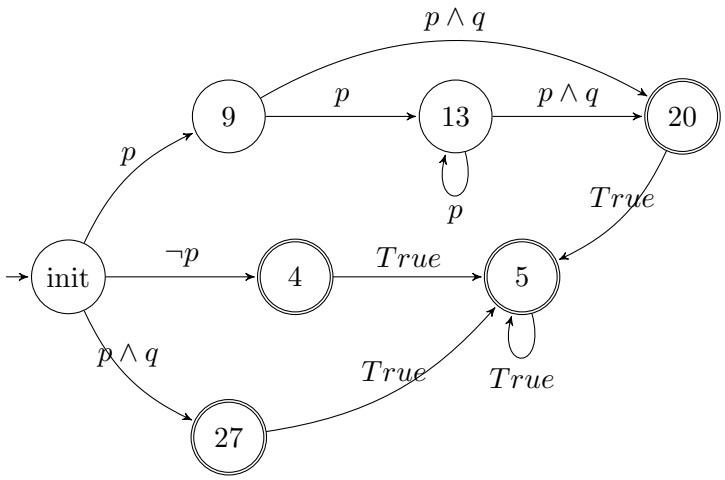
3. (60 points) Apply the simple on-the-fly translation algorithm to construct a generalized Büchi automaton from the PTL formula $p \to (p\,\mathcal{U}\,(p \wedge q))$. Please try to illustrate how the algorithm works by showing a few partially constructed automata during the translation.

Solution.

$p \to (p\,\mathcal{U}\,(p \wedge q)) \equiv \neg p \vee (p\,\mathcal{U}\,(p \wedge q))$


ID:1
Incoming:{init}
Old:∅
New:{¬p ∨ (p U (p ∧ q))}
Next:∅
split
split
ID:2
Incoming:{init}
Old:{¬p ∨ (p U (p ∧ q))}
New:{¬p}
Next:∅
ID:3
Incoming:{init}
Old:{¬p ∨ (p U (p ∧ q))}
New:{p U (p ∧ q)}
Next:∅
split
split
ID:4
Incoming:{init}
Old:{¬p ∨ (p U (p ∧ q)), ¬p}
New:∅
Next:∅
ID:7
Incoming:{init}
Old:{¬p ∨ (p U (p ∧ q)),
p U (p ∧ q)}
New:{p}
Next:{p U (p ∧ q)}
ID:8
Incoming:{init}
Old:{¬p ∨ (p U (p ∧ q)),
p U (p ∧ q)}
New:{p ∧ q}
Next:∅
create a new node
ID:5
Incoming:{4, 5, 20, 27}
Old:∅
New:∅
Next:∅
ID:9
Incoming:{init}
Old:{¬p ∨ (p U (p ∧ q)),
p U (p ∧ q), p}
New:∅
Next:p U (p ∧ q)
ID:25
Incoming:{init}
Old:{¬p ∨ (p U (p ∧ q)),
p U (p ∧ q), p ∧ q}
New:p, q
Next:∅
create a new node
create a new node
ID:6
Incoming:{5}
Old:∅
New:∅
Next:∅
Merge with ID:5
ID:10
Incoming:{9}
Old:∅
New:p U (p ∧ q)
Next:∅
ID:26
Incoming:{init}
Old:{¬p ∨ (p U (p ∧ q)),
p U (p ∧ q), p ∧ q, p}
New:q
Next:∅
α
β


α
β
split
split
ID:11
Incoming:{9}
Old:$p\,\mathcal{U}\,(p \wedge q)$
New:p
Next:$p\,\mathcal{U}\,(p \wedge q)$
ID:12
Incoming:{9}
Old:$p\,\mathcal{U}\,(p \wedge q)$
New:$p \wedge q$
Next:∅
ID:27
Incoming:{$init$}
Old:{$\neg p \vee (p\,\mathcal{U}\,(p \wedge q))$,
$p\,\mathcal{U}\,(p \wedge q), p \wedge q, p, q$}
New:∅
Next:∅
create a new node
ID:28
Incoming:{$init$}
Old:∅
New:∅
Next:∅
Merge with ID=5
ID:13
Incoming:{9, 13}
Old:$p\,\mathcal{U}\,(p \wedge q), p$
New:∅
Next:$p\,\mathcal{U}\,(p \wedge q)$
ID:22
Incoming:{9}
Old:$p\,\mathcal{U}\,(p \wedge q), p \wedge q$
New:p, q
Next:∅
create a new node
ID:14
Incoming:{13}
Old:∅
New:$p\,\mathcal{U}\,(p \wedge q)$
Next:∅
ID:23
Incoming:{9}
Old:$p\,\mathcal{U}\,(p \wedge q), p \wedge q, p$
New:q
Next:∅
split
split
ID:15
Incoming:{13}
Old:$p\,\mathcal{U}\,(p \wedge q)$
New:p
Next:$p\,\mathcal{U}\,(p \wedge q)$
ID:16
Incoming:{13}
Old:$p\,\mathcal{U}\,(p \wedge q)$
New:$p \wedge q$
Next:∅
ID:24
Incoming:{9}
Old:$p\,\mathcal{U}\,(p \wedge q), p \wedge q, p, q$
New:∅
Next:∅
Merge with ID=20
ID:17
Incoming:{13}
Old:$p\,\mathcal{U}\,(p \wedge q), p$
New:∅
Next:$p\,\mathcal{U}\,(p \wedge q)$
Merge with ID=13
ID:18
Incoming:{13}
Old:$p\,\mathcal{U}\,(p \wedge q), p \wedge q$
New:p, q
Next:∅
ID:19
Incoming:{13}
Old:$p\,\mathcal{U}\,(p \wedge q), p \wedge q, p$
New:q
Next:∅
ID:20
Incoming:{13, 9}
Old:$p\,\mathcal{U}\,(p \wedge q), p \wedge q, p, q$
New:∅
Next:∅
create a new node
ID:21
Incoming:{20}
Old:∅
New:∅
Next:∅
Merge with ID=5


$p \wedge q$
9
p
13
$p \wedge q$
20
p
p
$True$
init
$\neg p$
4
$True$
5
$p \wedge q$
$True$
$True$
27

□