Program Construction and Reasoning
Exercises (Part 3)

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Strengthening Invariants

1. Derive a solution for:

$$\begin{array}{l} |[\ \textbf{con}\ N : \mathit{int}\{N \geq 0\}; a : \textbf{array}\ [0..N)\ \textbf{of}\ \mathit{int}; \\ \quad \textbf{var}\ r : \mathit{int}; \\ \quad S \\ \quad \{r = (\uparrow i, j : 0 \leq i < j < N : a[i] - a[j])\} \\ ]|. \end{array}$$

2. Derive a solution for:

$$\begin{array}{l} |[\ \textbf{con}\ N : \mathit{int}\{N \geq 1\}; a : \textbf{array}\ [0..N)\ \textbf{of}\ \mathit{int}; \\ \quad \textbf{var}\ r : \mathit{int}; \\ \quad S \\ \quad \{r = (\# i, j : 0 \leq i < j < N : a[i] \times a[j] \geq 0)\} \\ ]|. \end{array}$$

Tail Invariants

3. Solve:

$$\begin{array}{l} |[\ \textbf{con}\ A, B : \mathit{int}\{A \geq 0\ \wedge B \geq 0\}; \\ \quad \textbf{var}\ r : \mathit{int}; \\ \quad S \\ \quad \{r = A \times B\} \\ ]|, \end{array}$$

   using only **div**2, **mod**2, $\times 2$, addition, and substraction.

4. The function *fusc* is defined on natural numbers by:

$$\begin{array}{lcl} \mathit{fusc}\ 0 & = & 0 \\ \mathit{fusc}\ 1 & = & 1 \\ \mathit{fusc}\ (2 \times n) & = & \mathit{fusc}\ n \\ \mathit{fusc}\ (2 \times n + 1) & = & \mathit{fusc}\ n + \mathit{fusc}\ (n + 1). \end{array}$$

   Derive a program computing $\mathit{fusc}\ N$ for $N \geq 0$. Hint: try $\mathit{fusc}\ 78$.