FLOLAC 2011

Decision Procedures and Logic Synthesis

Problem Set

Due on 2011/7/6 9:10am

LS 1 (Commutativity between Cofactor and Boolean Operations) (10%)
Given two Boolean functions f and g and a Boolean variable v, prove or disprove the following equalities:
(a) $(\neg f)_v = \neg(f_v)$
(b) $(f \ \vee \ g)_v = (f_v) \vee (g_v)$

LS 2 (Functional Decomposition) (40%)
Given a Boolean function f over variables $x_1, \ldots, x_6$, suppose $f(x_1, \ldots, x_6)$ can be expressed as the composition $h(x_1, x_2, x_3, g(x_4, x_5, x_6))$ of some functions g and h.
(a) What is the necessary and sufficient condition for f to be expressible with such decomposition? (Hint: Represent f in a table (similar to bi-decomposition analysis) with rows indexed by the truth assignments of variables (x_1, x_2, x_3) and columns by (x_4, x_5, x_6).)
(b) Please formulate the above decomposability condition as a satisfiability problem.
(c) Please formulate the computation of function g as an interpolation procedure from the above satisfiability formulation.
(d) How to compute function h?

LS 3 (Quantified Boolean Formula) (20%)
For Boolean functions f and g, show that
(a) $\forall x(f(x, y) \wedge g(x, z)) = \forall x f(x, y) \wedge \forall x g(x, z)$
(b) $\exists x(f(x, y) \wedge g(x, z)) \neq \exists x f(x, y) \wedge \exists x g(x, z)$
(c) $\neg \forall x f(x, y) = \exists x \neg f(x, y)$
(d) $\forall x \exists y f(x, y) \neq \exists y \forall x f(x, y)$

LS 4 (Quantifier Elimination) (20%, due on 7/7 9:10am)
Given an arbitrary quantified Boolean formula $\exists z f(x, y, z)$, suppose we would like to find some function $g(x, y)$ such that $f(x, y, g(x, y))$ equals $\exists z f(x, y, z)$. What is the condition for g in terms of f? (What are the smallest onset, smallest offset, and largest don't-care set of g?)