Program Construction and Reasoning
Exercises (Part 2)

Shin-Cheng Mu

2010 Formosan Summer School on
Logic, Language, and Computation
June 28 – July 9, 2010

Quantifications

1. An integer array $X[0..N)$ is given, where $N \geq 1$. Express the following sentences in a formal way:

   1. r is the sum of the elements of X.
   2. X is increasing.
   3. all values of X are distinct.
   4. r is the length of a longest constant segment of X.
   5. r is the maximum of the sums of the segments of X.

2. An integer array $X[0..N)$ is given, where $N \geq 1$. Express the following expressions in a natural language:

   1. $b \leftrightarrow (\forall i : 0 \leq i < N : X[i] \geq 0)$.
   2. $r = (\#k : 0 \leq k < N : (\forall i : 0 \leq i < k : X[i] < X[k]))$.
   3. $r = (\uparrow p, q : 0 \leq p \leq q \leq N \ \wedge \ (\forall i : p \leq i < q : X[i] > 0) : p - q)$.
   4. $r = (\#p, q : 0 \leq p < q < N : X[p] = 0 \ \wedge \ X[q] = 1)$.
   5. $s = (\uparrow p, q : 0 \leq p < q < N : X[p] + X[q])$.
   6. $b \leftrightarrow (\forall p, q : 0 \leq p \ \wedge \ 0 \leq q \ \wedge \ p + q = N - 1 : X[p] = X[q])$.

Taking Conjuncts as Invariants

3. Derive a program for the computation of square root.

$$
\begin{aligned}
&|[\ \mathbf{con} \ N : int\{N \geq 0\}; \\
&\quad \mathbf{var} \ x : int; \\
&\quad squareroot \\
&\quad \{x^2 \leq N \ \wedge \ (x+1)^2 > N\} \\
&]|
\end{aligned}
$$

Replacing Constants by Variables

4. Derive a solution for:

$$|[\textbf{ con } N : int\{N \geq 0\}; a : \textbf{array } [0..N) \textbf{ of } int;$$
$$\textbf{var } r : int;$$
$$S$$
$$\{r = (\uparrow i : 0 \leq i < N : a[i])\}$$
$$]|.$$

5. Derive a solution for:

$$|[\textbf{ con } N, X : int\{N \geq 0\}; a : \textbf{array } [0..N) \textbf{ of } int;$$
$$\textbf{var } r : int;$$
$$S$$
$$\{r = (\Sigma i : 0 \leq i < N : a[i] \times X^i)\}$$
$$]|.$$