Logic
Homework for Lecture II

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Please answer as many of the following questions as you can, in Chinese or English, on the provided answer sheet and hand it to me on or before **July 7, 2008**. No delayed submissions will be accepted.

Do not feel pressured to complete *all* questions. The grading of your homework will not be based on how many questions you solve, but on how well you do compared with your classmates.

1 Sequent Calculus for Propositional Logic

Give derivations of the following sequents:

1. $\vdash (P \land Q \to R) \to (P \to Q \to R)$
2. $\vdash P \to \neg\neg P$
3. $\vdash (P \to Q) \lor (Q \to P)$
4. $\vdash P \lor \neg P$
5. $\vdash \neg(P \land Q) \leftrightarrow \neg P \lor \neg Q$

2 Sequent Calculus for First-order Logic

Hint: To solve the problems below, you may make use of the fact that $\varphi[x/x] \equiv \varphi$ for any formula φ and any variable x.

1. Give a derivation of $\vdash \varphi \leftrightarrow (\forall x.\varphi)$, where φ is a formula such that $x \notin \mathrm{FV}(\varphi)$. Which part of the derivation fails when this condition is not satisfied?
2. Can you give a derivation of $\vdash (\forall x.\varphi) \to (\exists x.\varphi)$ for any formula φ? Would you accept this inference step in a mathematical proof? Why or why not?
3. Show that $\vdash_{\mathrm{LK}} \neg(\exists x.\neg\varphi) \to (\forall x.\varphi)$ for any formula φ.
4. Show that $\vdash_{\mathrm{LK}} (\exists x.\forall y.\varphi) \to (\forall y.\exists x.\varphi)$ for any formula φ.

   Give a structure $\mathcal{M}$ and a formula φ with free variables x and y such that $\mathcal{M} \models \forall y.\exists x.\varphi$, but $\mathcal{M} \not\models \exists x.\forall y.\varphi$.