Logic

Homework for Lecture I

Max Schäfer

Please answer as many of the following questions as you can, in Chinese or English, on the provided answer sheet and hand it to me on or before **July 2, 2010**. No delayed submissions will be accepted.

Do not feel pressured to complete *all* questions. The grading of your homework will not be based on how many questions you solve, but on how well you do compared with your classmates.

1. Show the following equivalence (often called *Consensus Theorem*) by drawing up a truth table:

$$P \wedge Q \;\vee\; \neg P \wedge R \;\vee\; Q \wedge R \;\Leftrightarrow\; P \wedge Q \;\vee\; \neg P \wedge R$$

2. The connective $\overline{\vee}$ ("nor") is defined by $\varphi \overline{\vee} \psi := \neg(\varphi \vee \psi)$.

   (a) Draw a truth table for $P \overline{\vee} Q$.

   (b) Find formulas $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ with $\overline{\vee}$ as their only connective such that

      i. $\varphi_1 \Leftrightarrow \neg P$

      ii. $\varphi_2 \Leftrightarrow P \vee Q$

      iii. $\varphi_3 \Leftrightarrow P \wedge Q$

      iv. $\varphi_4 \Leftrightarrow P \to Q$

      You do not have to provide truth tables.

   Is $\{\overline{\vee}\}$ a functionally complete set?

3. Prove the following tautologies using calculational reasoning with the laws on pg. 12 of the lecture notes (you may additionally want to use the law $P \vee \bot \leftrightarrow P$ proved in the lecture):

   (a) $P \wedge P \leftrightarrow P$

   (b) $P \wedge Q \leftrightarrow Q \wedge P$

   (c) (*extra credit*) $P \wedge (Q \wedge R) \leftrightarrow (P \wedge Q) \wedge R$

   (d) $\neg(P \vee Q) \leftrightarrow \neg P \wedge \neg Q$ (Hint: start with the right hand side)

4. (*extra credit*) Back on the island of knights and knaves, inhabitant A says "It is not the case that I am a knight if B says so." Using calculational logic, what can you deduce about A and B?