

## Homework Assignment 3

[Compiled on June 28, 2009]

### Note

This assignment is due 9AM, July 6, 2009. Please write or type your answers on A4 (or similar size) paper. Put your completed homework by the due time on the lecturer's desk in Room 207 (of the NTU Extension). **No late submission will be accepted. You may discuss the problems with others, but copying answers is strictly forbidden.**

### Problems

1. Show that the following properties of distributed knowledge are valid:
  - (a)  $(D_G\varphi \wedge D_G(\varphi \supset \psi)) \supset D_G\psi$ ,
  - (b)  $D_G\varphi \supset \varphi$  (assuming that the  $\mathcal{K}_i$  relations are all reflexive),
  - (c)  $D_G\varphi \supset D_G D_G\varphi$  (assuming that the  $\mathcal{K}_i$  relations are all transitive),
  - (d)  $\neg D_G\varphi \supset D_G\neg D_G\varphi$  (assuming that the  $\mathcal{K}_i$  relations are all symmetric and transitive),
  - (e)  $D_{\{i\}}\varphi \equiv K_i\varphi$ ,
  - (f)  $D_G\varphi \supset D_{G'}\varphi$  if  $G \subseteq G'$ .
2. Let  $\alpha^-$  denote the converse action of  $\alpha$  and define its state transition relation as  $R_{\alpha^-} = \{(w, u) \mid (u, w) \in R_\alpha\}$ . Then, prove the following PDL wffs are valid (recall that we write  $\alpha; \beta$  as  $\alpha\beta$ ):
  - (a)  $\langle \alpha \rangle \varphi \supset \langle \alpha\alpha^- \rangle \varphi$ ,
  - (b)  $\langle \alpha^*\alpha \rangle \varphi \equiv \langle \alpha\alpha^* \rangle \varphi$ ,
  - (c)  $\langle a(ba)^* \rangle \varphi \equiv \langle (ab)^*a \rangle \varphi$ , where  $a$  and  $b$  are atomic programs.