

Homework Assignment 1

[Compiled on June 28, 2009]

Note

This assignment is due 9AM, July 3, 2009. Please write or type your answers on A4 (or similar size) paper. Put your completed homework by the due time on the lecturer's desk in Room 207 (of the NTU Extension). **No late submission will be accepted. You may discuss the problems with others, but copying answers is strictly forbidden.**

Problems

1. According to Aristotle's logical square of opposition, $\Box p$ and $\Box \neg p$ is in contrary relation. In other words, $\Box p \wedge \Box \neg p$ is unsatisfiable. However, this is wrong according to Kripke semantics. Please provide a model $\mathfrak{M} = (W, R, \pi)$ and a world $w \in W$ such that $\mathfrak{M}, w \Vdash \Box p \wedge \Box \neg p$.
2. A model $\mathfrak{M} = (W, R, \pi)$ satisfies *secondary reflexivity* if for any $w, u \in W$, if $(w, u) \in R$, then $(u, u) \in R$. Prove that $\Box(\Box \varphi \supset \varphi)$ is valid in all secondary reflexive models.
3. (a) Find a model $\mathfrak{M} = (W, R, \pi)$ and a world $w \in W$ such that $\mathfrak{M}, w \Vdash p$ and $\mathfrak{M}, w \not\Vdash \Box p$. Conclude that $\{p\} \not\Vdash_{\mathcal{C}} \Box p$ if \mathcal{C} is the class of all Kripke models.
 (b) Prove that $\{p\} \Vdash_{\mathcal{C}}^g \Box p$ for any class of all Kripke models \mathcal{C} .
4. Let $\overleftarrow{\Box}$ denote a backward looking modality with the following satisfaction condition:

$$\mathfrak{M}, w \Vdash \overleftarrow{\Box} \varphi \Leftrightarrow \text{there exists } u \in W \text{ such that } (u, w) \in R \text{ and } \mathfrak{M}, u \Vdash \varphi.$$

Use generated submodels to show that $\overleftarrow{\Box}$ is not definable in basic modal logic.

5. Prove that a bounded morphism is a bisimulation, i.e. if $f : \mathfrak{M} \rightarrow \mathfrak{M}'$, then $\mathfrak{M}, w \leftrightarrow \mathfrak{M}', f(w)$ for all w in \mathfrak{M} .