FLOLAC 2009

Hardware Equivalence and Property Verification

Problem Set

Due on 2009/7/7 9:10am

EC 1 (Commutativity between Cofactor and Boolean Operations) (10pt)
Given two Boolean functions f and g and a Boolean variable v, prove or disprove the following equalities:
(a) $(\neg f)_v = \neg(f_v)$
(b) $(f \ \vee \ g)_v = (f_v) \vee (g_v)$

EC 2 (Quantified Boolean Formula) (20pt)
For Boolean functions f and g, show that
(a) $\forall x(f(x,y) \wedge g(x,z)) = \forall x f(x,y) \wedge \forall x g(x,z)$
(b) $\exists x(f(x,y) \wedge g(x,z)) \neq \exists x f(x,y) \wedge \exists x g(x,z)$
(c) $\neg \forall x f(x,y) = \exists x \neg f(x,y)$
(d) $\forall x \exists y f(x,y) \neq \exists y \forall x f(x,y)$

EC 3 (Reachability Analysis) (30pt)
Given the two FSMs M_1 and M_2 with state transition graphs of Slide 46, let

- x be the input variable,
- y_1 and y_2 be the output variables of M_1 and M_2, respectively, and
- p (p') be the current-state (next-state) variable of M_1, and q_1 and q_2 (q_1' and q_2') be the current-state (next-state) variables of M_2.

Suppose we encode state s_0 as 0 (i.e., with characteristic function $\neg p$), s_1 as 1 (i.e., p), state t_0 as 00 (i.e., $\neg q_1 \neg q_2$), state t_1 as 01 (i.e., $\neg q_1 q_2$), state t_2 as 10 (i.e., $q_1 \neg q_2$), state t_3 as 11 (i.e., $q_1 q_2$).

(a) What is the state-transition function δ of M_1? What are the state-transition functions δ_1 and δ_2 of M_2?
(b) What is the transition relation $T_\exists$ of the product machine?
(c) What is the output function of the product machine?
(d) What is the characteristic function of the initial state of the product machine? (Assume s_0 (t_0) is the initial state of M_1 (M_2).)
(e) Compute the characteristic function of the "bad" states of the product machine. (Write down the quantified Boolean formula as well as its quantifier-free formula after quantification.)
(f) Compute the characteristic function of the one-step reachable states from the initial state using image computation.

EC 4 (Existential Quantification) (due on 7/8)
Given an arbitrary quantified Boolean formula $\exists z f(x,y,z)$, suppose we would like to find some function $g(x,y)$ such that $f(x,y,g(x,y))$ equals $\exists z f(x,y,z)$. What is the condition for g in terms of f? (What are the smallest onset, smallest offset, and largest don't-care set of g?)