Glossary for Logic

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Connectives and Quantifiers

$\bot$	falsity
$\top$	truth
$\varphi \land \psi$	conjunction, "and"
$\varphi \lor \psi$	disjunction, "or"
$\varphi \to \psi$	implication, "if…then…"
$\neg\varphi$	negation, "not"
$\varphi \leftrightarrow \psi$	equivalence
$\forall x.\varphi$	universal quantification, "for all individuals x, φ"
$\exists x.\varphi$	existential quantification, "there is an individual x such that φ"
$\forall P.\varphi$	(2nd order) universal quantification, "for all propositions P, φ"
$\exists P.\varphi$	(2nd order) existential quantification, "there is a proposition P such that φ"

Semantic Definitions

valid	a formula that is always true
satisfiable	a formula that has a model
$\varphi \Leftrightarrow \psi$	φ and ψ have same truth value under all interpretations
$\Gamma \models \varphi$	any model for Γ is also a model for φ

NJ

Propositional Rules

Conjunction Introduction:

$$(\land\mathrm{I})\ \frac{\varphi \qquad \psi}{\varphi \land \psi}$$

Conjunction Elimination:

$$(\land\mathrm{E}_l)\ \frac{\varphi \land \psi}{\varphi} \qquad\qquad (\land\mathrm{E}_r)\ \frac{\varphi \land \psi}{\psi}$$

Disjunction Introduction:

$$(\lor\mathrm{I}_l)\ \frac{\varphi}{\varphi \lor \psi} \qquad\qquad (\lor\mathrm{I}_r)\ \frac{\psi}{\varphi \lor \psi}$$

Disjunction Elimination:

$$(\lor\mathrm{E}^{v,w})\ \frac{\varphi \lor \psi \qquad \begin{array}{c}[v\colon\varphi]\\ \vdots \\ \vartheta\end{array} \qquad \begin{array}{c}[w\colon\psi]\\ \vdots \\ \vartheta\end{array}}{\vartheta}$$

All open assumptions from the left subderivation are also open in the two right subderivations.

Implication Introduction:

$$(\to\mathrm{I}^{x})\ \frac{\begin{array}{c}[x\colon\varphi]\\ \vdots \\ \psi\end{array}}{\varphi \to \psi}$$

Implication Elimination (*modus ponens*, MP):

$$(\to\mathrm{E})\ \frac{\varphi \to \psi \qquad \varphi}{\psi}$$

Falsity Elimination (EFQ):

$$(\bot\text{E})\ \frac{\bot}{\varphi}$$

First Order Rules

Universal Introduction:

$$(\forall\text{I})\ \frac{\varphi}{\forall x.\varphi}$$

where x cannot occur free in any open assumption

Universal Elimination:

$$(\forall\text{E})\ \frac{\forall x.\varphi}{\varphi[t/x]}$$

for any term t

Existential Introduction:

$$(\exists\text{I})\ \frac{\varphi[t/x]}{\exists x.\varphi}$$

for any term t

Existential Elimination:

$$(\exists\text{E}^u)\ \frac{\exists x.\varphi \qquad \begin{array}{c}[u\colon \varphi]\\ \vdots \\ \psi\end{array}}{\psi}$$

where x cannot occur free in any open assumptions on the right and in ψ

All open assumptions from the left subderivation are also open in the right subderivation.

NK

NK has all rules of NJ except $(\bot\text{E})$. Instead it has the rule

$$(\text{DN})\ \frac{\neg\neg\varphi}{\varphi}$$

NJ^2

NJ^2 has all propositional rules of NJ, plus the following:

Universal Introduction:

$$(\forall\text{I})\ \frac{\varphi}{\forall P.\varphi}$$

where P cannot occur free in any open assumption

Universal Elimination:

$$(\forall\text{E})\ \frac{\forall P.\varphi}{\varphi[\psi/P]}$$

for any formula ψ

Existential Introduction:

$$(\exists\text{I})\ \frac{\varphi[\psi/P]}{\exists P.\varphi}$$

for any formula ψ

Existential Elimination:

$$(\exists\text{E}^u)\ \frac{\exists P.\varphi \qquad \begin{array}{c}[u\colon \varphi]\\ \vdots \\ \psi\end{array}}{\psi}$$

where P cannot occur free in any open assumptions on the right and in ψ

All open assumptions from the left subderivation are also open in the right subderivation.