

Logic

Homework for Lecture III

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Please answer as many of the following questions as you can, in Chinese or English, on the provided answer sheet and hand it to me on or before **July 10, 2008**. No delayed submissions will be accepted.

Do not feel pressured to complete *all* questions. The grading of your homework will not be based on how many questions you solved, but on how well you did compared with your classmates.

1 Logic, Semantics, and Deductive Systems

Show that

- $\vdash_{\text{NJ}} \neg\neg\neg\neg P \rightarrow \neg\neg P$ for any propositional letter P .
- $u: \neg\neg\varphi \rightarrow \varphi, v: \neg\neg\psi \rightarrow \psi \vdash_{\text{NJ}} \neg\neg(\varphi \wedge \psi) \rightarrow \varphi \wedge \psi$
- Compute $((P \rightarrow Q) \rightarrow P) \rightarrow P$.*
Bonus: Show that $\vdash_{\text{NJ}} (((P \rightarrow Q) \rightarrow P) \rightarrow P)^*$

2 Proof Normalization (Bonus)

Recall the Church encoding of natural numbers as lambda terms: The number n is encoded as the term $\lambda s: A \rightarrow A. \lambda z: A. s^n z$, where $s^n z$ represents the n -fold application of s to z (for details, see Dr. Chen's lecture notes).

The successor function S for Church numerals can be implemented by the lambda term

$$\lambda x: (A \rightarrow A) \rightarrow A \rightarrow A. \lambda s: A \rightarrow A. \lambda z: A. x s (s z)$$

Give a derivation corresponding to the term $S 0$ and normalize it. What proof term does the resulting derivation correspond to?

3 Curry-Howard

Show that there is no simply typed lambda term M such that $\vdash M: (A \rightarrow A) \rightarrow A$.