

Logic

Homework for Lecture I

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Please answer as many of the following questions as you can, in Chinese or English, on the provided answer sheet and hand it to me on or before **July 4, 2008**. No delayed submissions will be accepted.

Do not feel pressured to complete *all* questions. The grading of your homework will not be based on how many questions you solve, but on how well you do compared with your classmates.

1 Propositional Logic

1. Prove the following using truth tables or the definition of validity and equivalence:

(a) $P \wedge Q \rightarrow R \Leftrightarrow P \rightarrow Q \rightarrow R$

(b) $\models \perp \rightarrow P$

(c) $P \wedge Q \Leftrightarrow \neg(P \rightarrow \neg Q)$

(d) $P \rightarrow Q \Leftrightarrow \neg P \vee Q$

2. Show Neutrality and Boundedness (slide 27) using only the Important Equivalences, Idempotency, and algebraic reasoning.

3. The connective $\bar{\wedge}$ (“nand”) is defined by

$$\varphi \bar{\wedge} \psi := \neg(\varphi \wedge \psi)$$

- (a) Draw a truth table for $P \bar{\wedge} Q$.

- (b) Find formulas $\varphi_1, \varphi_2, \varphi_3$ with $\bar{\wedge}$ as their only connective such that

i. $\varphi_1 \Leftrightarrow \neg P$

ii. $\varphi_2 \Leftrightarrow P \wedge Q$

iii. $\varphi_3 \Leftrightarrow \perp$

You do not have to provide truth tables.

Thus, $\{\bar{\wedge}\}$ is a functionally complete set!

2 First Order Logic

1. Can you find a signature in which $\forall x.\forall y.r(x, y) \wedge s(x) \rightarrow (\exists y.r(y))$ is a formula?
2. Evaluate the following substitutions, indicating where you need alpha equivalence:
 - $(y \approx x \vee x \approx x \wedge (\forall y.x < y))[y/x]$
 - $(\forall u.\forall v.p(u) \rightarrow q(x) \wedge (\exists z.p(x) \wedge (\forall x.q(x, z))))[x/z]$
3. Prove that $(\forall x.\varphi) \Leftrightarrow \neg(\exists x.\neg\varphi)$ for any formula φ .