Logic

Homework for Lecture III

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Please answer as many of the following questions as you can, in Chinese or English, on the provided answer sheet and hand it to me before noon on **July 13, 2007**. No delayed submissions will be accepted. *None.*

Do not feel pressured to complete *all* questions. The grading of your homework will not be based on how many questions you solved, but on how well you did compared with your classmates.

1 Equational Logic

1. Prove, for any terms r, s, variable x, interpretation I and variable assignment σ, that $[\![[x := r]s]\!]_{I,\sigma} = [\![s]\!]_{I,\sigma[x := [\![r]\!]_{I,\sigma}]}$.

2. Prove the (semantic) validity of the closure law.

3. Prove the transitivity law in NJEq (you may assume $\Gamma = \emptyset$).

4. Prove the closure law in NJEq.

   Hint: You may first want to prove (syntactically) that

   (a) $\mathrm{FV}([x := s]r) \subseteq \mathrm{FV}(r) \cup \mathrm{FV}(s)$

   (b) if $y \notin \mathrm{FV}(s)$, then $[y := t]s \equiv s$

   (c) if $y \notin \mathrm{FV}(r)$, then $[y := s][x := y]r \equiv [x := s]r$

   for terms r, s, t and variables x, y.

5. Prove that the equality $x \sqcap 1 = x$ holds in every Boolean algebra...

   (a) ...using the definition of a Boolean algebra.

   (b) ...using the theory of Boolean algebras in NJEq.

2 Curry-Howard (Bonus)

1. Give an annotated derivation corresponding to the lambda term $\lambda x\colon a.\lambda y\colon b.x$.
2. Give an annotated derivation corresponding to the lambda term

   $$\lambda m\colon (a \to a) \to a \to a.\lambda n\colon (a \to a) \to a \to a.\lambda s\colon a \to a.\lambda z\colon a.m\, s\, (n\, s\, z)$$