Logic

Homework for Lecture II

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Please answer as many of the following questions as you can, in Chinese or English, on the provided answer sheet and hand it to me before noon on **July 6, 2007**. No delayed submissions will be accepted. *None.*

Do not feel pressured to complete *all* questions. The grading of your homework will not be based on how many questions you solved, but on how well you did compared with your classmates.

1 Natural Deduction for Propositional Logic

1. Give a derivation of $a \wedge b \to a \vee b$.
2. Give a derivation of $a \to a \wedge a$.
3. Give a derivation of $a \vee (b \vee c) \to (a \vee b) \vee c$.
4. Give derivations of $a \vee (a \wedge b) \to a$ and $a \to a \vee (a \wedge b)$.
5. Give a derivation of $\neg a \wedge \neg b \to \neg(a \vee b)$.
6. Give a derivation of $\neg p \to \neg\neg\neg p$.

2 Natural Deduction for First Order Logic

1. Give a derivation of $\varphi \leftrightarrow (\forall x.\varphi)$, where φ is a formula such that $x \notin \mathrm{FV}(\varphi)$. Which part of the derivation fails when the last condition is not satisfied?
2. Can you give a derivation of $(\forall x.\varphi) \to (\exists x.\varphi)$ for any formula φ? Would you accept this inference step in a mathematical proof? Why or why not?
3. Show that $\forall x.\varphi \wedge \psi \vdash_{\mathrm{NJ}} (\forall x.\varphi) \wedge (\forall x.\psi)$ for any formulas φ and ψ.
4. Show that $(\exists x.\varphi) \vee (\exists x.\psi) \vdash_{\mathrm{NJ}} (\exists x.\varphi \vee \psi)$ for any formulas φ and ψ.