Logic

Homework for Lecture I

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Please answer as many of the following questions as you can, in Chinese or English, on the provided answer sheet and hand it to me in class on **July 3, 2007**. No delayed submissions will be accepted. *None.*

Do not feel pressured to complete *all* questions. The grading of your homework will not be based on how many questions you solved, but on how well you did compared with your classmates.

1 Propositional Logic

1. Compute $\mathrm{Sub}((\neg a \vee b \wedge a) \rightarrow (b \leftrightarrow \neg\neg(a \vee a)))$.
2. Show the following equivalence (often called *Consensus Theorem*):

$$(a \vee b) \wedge (\neg a \vee c) \wedge (b \vee c) = (a \vee b) \wedge (\neg a \vee c)$$

3. Show idempotency of $\wedge$ and $\vee$ using only the "Important Equivalences".
4. Prove Lemma 2.3.
5. Give a canonical DNF of $(a \vee b \vee \neg c) \wedge (c \vee a) \wedge b$.
6. Express $\neg$ and $\wedge$ in terms of nor.

2 First Order Logic

1. Can you find a signature in which $\forall x.\forall y.r(x, y) \wedge s(x) \rightarrow (\exists y.r(y))$ is a formula?
2. Give NNFs of the following formulas:
   - $\neg(\exists x.(\forall y.r(x, y)) \rightarrow (\exists z.k(z)) \wedge (\forall a.k(x)))$
   - $\forall x.\neg(p(x) \leftrightarrow (q(y) \rightarrow r))$
   - $\neg(\exists z.h(z) \vee \neg(\exists y.l(y)))$

3. Evaluate the following substitutions, indicating where you need alpha equivalence:

   - $[x := s(s(0))](y \approx x \vee x \approx x \wedge (\forall x. x < y))$
   - $[x := z](\forall u. \forall v. p(u) \rightarrow q(x) \wedge (\exists z. p(x) \wedge (\forall x. q(x, z))))$

4. Prove that $(\exists x. \varphi \vee \psi) = (\exists x. \varphi) \vee (\exists x. \psi)$ for any two formulas φ and ψ.

5. Prove that $(\exists x. \varphi \wedge \psi) = \varphi \wedge (\exists x. \psi)$ for two formulas φ and ψ if $x \notin \mathrm{FV}(\varphi)$.

6. Give a PNF of $\neg(\exists x. (\forall y. r(x, y)) \rightarrow (\exists x. k(x)) \wedge (\forall a. k(y)))$.