Logical Relations for Capabilities

Neel Krishnaswami
University of Cambridge

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An Effectful Modal Language

$$A ::= 1 \mid A \to B \mid \mathbb{N} \mid \mathrm{Chan} \mid \Box A$$

$$\begin{aligned} e ::= {} & () \mid \lambda x{:}A.\,e \mid e\ e' \mid n \mid x \mid \mathrm{print}(e,e') \\ \mid {} & \mathrm{let}\ x = e_1\ \mathrm{in}\ e_2 \\ \mid {} & \mathrm{box}(e) \mid \mathrm{let}\ \mathrm{box}(x) = e_1\ \mathrm{in}\ e_2 \end{aligned}$$

$$\Gamma ::= \cdot \mid \Gamma, x : A \qquad \Delta ::= \cdot \mid \Delta, x : A$$

$$\Delta ; \Gamma \vdash e : A$$

Typing Rules

$$\frac{}{\Delta;\Gamma \vdash (): 1} \qquad \frac{}{\Delta;\Gamma \vdash n : \mathbb{N}}$$

$$\frac{\Delta;\Gamma, x:A \vdash e : B}{\Delta;\Gamma \vdash \lambda x{:}A.e : A \to B} \qquad \frac{\Delta;\Gamma \vdash e : A \to B \quad \Delta;\Gamma \vdash e' : A}{\Delta;\Gamma \vdash e\ e' : B}$$

$$\frac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e, e') : 1}$$

$$\frac{\Delta;\Gamma \vdash e_1 : A \qquad \Delta;\Gamma, x:A \vdash e_2 : C}{\Delta;\Gamma \vdash \mathrm{let}\ x = e_1\ \mathrm{in}\ e_2 : C} \qquad \frac{x:A \in \Gamma}{\Delta;\Gamma \vdash x : A}$$

Modal Typing Rules

$$\frac{x : A \in \Delta}{\Delta ; \Gamma \vdash \underline{x} : A}$$

$$\frac{\Delta ; \cdot \vdash e : A}{\Delta ; \Gamma \vdash \mathrm{box}(e) : \Box A}$$

$$\frac{\Delta ; \Gamma \vdash e_1 : \Box A \qquad \Delta , x : A ; \Gamma \vdash e_2 : C}{\Delta ; \Gamma \vdash \mathrm{let}\ \mathrm{box}(x) = e_1\ \mathrm{in}\ e_2 : C}$$

Reduction Rules, with Box

$$\frac{e \xrightarrow{a} e'}{\mathrm{box}(e) \xrightarrow{a} \mathrm{box}(e')}$$

$$\frac{}{\mathrm{let}\ \mathrm{box}(x) = \mathrm{box}(v)\ \mathrm{in}\ e \longrightarrow [v/x]e}$$

$$\frac{}{(\lambda x : A . e)\ v \longrightarrow [v/x]e}$$

$$\frac{}{\mathrm{print}(c, n) \xrightarrow{wr(c,n)} ()}$$

$$\frac{e_1\ e_2 \xrightarrow{a} e_1'\ e_2}{(e_1\ e_2) \xrightarrow{a} (e_1'\ e_2)}$$

$$\frac{e_2 \xrightarrow{a} e_2'}{v\ e_2 \xrightarrow{a} v\ e_2'}$$

$$\frac{}{\mathrm{let}\ x = v\ \mathrm{in}\ e_2 \longrightarrow [v/x]e_2}$$

$$\frac{e_1 \xrightarrow{a} e_1'}{\mathrm{let}\ x = e_1\ \mathrm{in}\ e_2 \xrightarrow{a} \mathrm{let}\ x = e_1'\ \mathrm{in}\ e_2}$$

What are we proving?

If $\bullet;\bullet \vdash e : A$ then $e \overset{\bullet}{\longrightarrow}{}^* v$

$$\boxed{\text{A closed term never writes}}$$

Preliminaries

If $e \xrightarrow{w}{}^{*} e'$ then chans(w) are the channel names in w

$$\begin{aligned}
\text{chans}(\cdot) &= \emptyset \\
\text{chans}(w_r(c,n)) &= \{c\} \\
\text{chans}(w \cdot w') &= \text{chans}(w) \cup \text{chans}(w')
\end{aligned}$$

Let $\mathcal{C}, \mathcal{C}'$ be a subset of channel names.

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$$

Let $\mathcal{C}, \mathcal{C}$ be a subset of channel names.

A Kripke Logical Relation

$$V_1^C = \{ () \}$$

$$V_{\mathbb{N}}^C = \{ n \mid n \in \mathbb{N} \}$$

$$V_{\mathrm{Chan}}^C = \{ c \mid c \in C \} = C$$

$$V_{A \to B}^C = \{ v \mid \forall C' \supseteq C,\ v' \in V_A^{C'}.\ v\,v' \in \mathcal{E}_B^{C'} \}$$

$$V_{\Box A}^C = \{ \mathrm{box}(v) \mid v \in V_A^{\phi} \}$$

$$\mathcal{E}_A^C = \{ e \mid e \xrightarrow{w}{}^{*} v,\ \mathrm{chans}(w) \subseteq C,\ v \in V_A^C \}$$

$\boxed{\text{A channel set AND type-indexed family of sets of terms}}$

A Kripke Frame

$$\left(W = \mathcal{P}(\mathrm{Chan}),\ \subseteq \right)$$

$$\mathcal{E}^{C}_{A} = \{ e \mid e \xrightarrow{w}{}^{*} v,\ \mathrm{chans}(w) \subseteq C,\ v \in V^{C}_{A} \}$$

$$\mathrm{safe\_print} : \Box(\underbrace{\mathrm{Chan} \to \mathbb{N} \to 1}_{f \,\in\, V^{\emptyset}_{\mathrm{Chan} \to \mathbb{N} \to 1}})$$

$\mathcal{C}$ - Closure

If $e \xrightarrow{a} e'$ and $chans(a) \subseteq \mathcal{C}$ then $e \in \mathcal{C}^{C}_{A}$ iff $e' \in \mathcal{C}^{C}_{A}$

$\mathcal{C}$-Closure

If $e \xrightarrow{a} e'$ and $chans(a) \subseteq \mathcal{C}$ then $e \in \mathcal{E}_A^{\mathcal{C}}$ iff $e' \in \mathcal{E}_A^{\mathcal{C}}$

Proof: Assume $e \xrightarrow{a} e'$ and $chans(a) \subseteq \mathcal{C}$

$\Leftarrow$ Assume $e' \in \mathcal{E}_A^{\mathcal{C}}$

By definition $e' \xrightarrow{w}{}^{*} v$, $v \in V_A^{\mathcal{C}}$, $chans(w) \subseteq \mathcal{C}$

By transitity, $e \xrightarrow{a \cdot w}{}^{*} v$

Since $chans(a) \subseteq \mathcal{C}$ and $chans(w) \subseteq \mathcal{C}$, $chans(a \cdot w) \subseteq \mathcal{C}$

Hence $e \in \mathcal{E}_A^{\mathcal{C}}$

$\mathcal{C}$-Closure

If $e \xrightarrow{a} e'$ and $\text{chans}(a) \subseteq \mathcal{C}$ then $e \in \mathcal{E}_A^{\mathcal{C}}$ iff $e' \in \mathcal{E}_A^{\mathcal{C}}$

Proof: Assume $e \xrightarrow{a} e'$ and $\text{chans}(a) \subseteq \mathcal{C}$

$\Rightarrow$ Assume $e \in \mathcal{E}_A^{\mathcal{C}}$

Hence $e \xrightarrow{w}{}^{*} v$, $\text{chans}(w) \subseteq \mathcal{C}$, $v \in V_A^{\mathcal{C}}$

By determinacy, $e \xrightarrow{a} e' \xrightarrow{w'}{}^{*} v$ s.t. $w = a \cdot w'$

Since $\text{chans}(a \cdot w') \subseteq \mathcal{C}$, $\text{chans}(w') \subseteq \mathcal{C}$

Hence $e' \in \mathcal{E}_A^{\mathcal{C}}$

Kripke Monotonicity

If $C \subseteq C'$ then

1. $V_A^{C} \subseteq V_A^{C'}$
2. $\mathcal{E}_A^{C} \subseteq \mathcal{E}_A^{C'}$

Proof by induction on types

Adding more allowed channels, allows more programs

Kripke Monotonicity

If $\mathcal{C} \subseteq \mathcal{C}'$ then $V_A^{\mathcal{C}} \subseteq V_A^{\mathcal{C}'}$ and $\mathcal{E}_A^{\mathcal{C}} \subseteq \mathcal{E}_A^{\mathcal{C}'}$

Case: $A \to B$

1. Assume $v \in V_{A \to B}^{\mathcal{C}}$
2. Hence $\forall \mathcal{C}'' \supseteq \mathcal{C}, v' \in V_A^{\mathcal{C}''}.\ v\ v' \in \mathcal{E}_B^{\mathcal{C}''}$

WTS $\forall \mathcal{C}'' \supseteq \mathcal{C}', v' \in V_A^{\mathcal{C}''}.\ v\ v' \in \mathcal{E}_B^{\mathcal{C}''}$

3. Assume $\mathcal{C}'' \supseteq \mathcal{C}'$, $v' \in V_A^{\mathcal{C}''}$
4. Since $\mathcal{C}' \supseteq \mathcal{C}$, $\mathcal{C}'' \supseteq \mathcal{C}$.
5. By 2, 4, 3. $v\ v' \in \mathcal{E}_B^{\mathcal{C}''}$
6. $v \in V_{A \to B}^{\mathcal{C}'}$

Kripke Monotonicity

If $\mathcal{C} \subseteq \mathcal{C}'$ then $V_A^{\mathcal{C}} \subseteq V_A^{\mathcal{C}'}$ and $\mathcal{E}_A^{\mathcal{C}} \subseteq \mathcal{E}_A^{\mathcal{C}'}$

Case: $A \to B$

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3. Assume $\mathcal{C}'' \supseteq \mathcal{C}'$, $v' \in V_A^{\hat{\mathcal{C}}}$
4. Since $\mathcal{C}' \supseteq \mathcal{C}$, $\mathcal{C}'' \supseteq \mathcal{C}$.
5. By 2, 4, 3. $v\ v' \in \mathcal{E}_B^{\mathcal{C}''}$
6. $v \in V_{A \to B}^{\mathcal{C}'}$

Kripke Monotonicity

If $\mathcal{C} \subseteq \mathcal{C}'$ then $V_A^{\mathcal{C}} \subseteq V_A^{\mathcal{C}'}$ and $\mathcal{E}_A^{\mathcal{C}} \subseteq \mathcal{E}_A^{\mathcal{C}'}$

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1. Assume $v \in V_{A \to B}^{\mathcal{C}}$
2. Hence $\forall \mathcal{C}'' \supseteq \mathcal{C}, v' \in V_A^{\mathcal{C}''}.\ v\, v' \in \mathcal{E}_B^{\mathcal{C}''}$

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3. Assume $\mathcal{C}'' \supseteq \mathcal{C}'$, $v' \in V_A^{\hat{\mathcal{C}}}$
4. Since $\mathcal{C}' \supseteq \mathcal{C}$, $\mathcal{C}'' \supseteq \mathcal{C}$.
5. By 2, 4, 3. $v\, v' \in \mathcal{E}_B^{\mathcal{C}''}$
6. $v \in V_{A \to B}^{\mathcal{C}'}$

Lifting to Substitutions

$$V_\Delta \triangleq \begin{cases} V_{\cdot} = \{ [\,] \} \\ V_{\Delta, x:A} = \{ (\delta, v/x) \mid \delta \in V_\Delta, v \in V_A^{\phi} \} \end{cases}$$

$$V_\Gamma^c \triangleq \begin{cases} V_{\cdot}^c = \{ [\,] \} \\ V_{\Gamma, x:A}^c = \{ (\gamma, v/x) \mid \gamma \in V_\Gamma^c, v \in V_A^c \} \end{cases}$$

$$V_{\Delta;\Gamma}^c = \{ [\delta ; \gamma] \mid \delta \in V_\Delta, \gamma \in V_\Gamma^c \}$$

Fundamental Lemma

If $\Delta ; \Gamma \vdash e : A$ and $[\delta ; \gamma] \in \mathcal{V}^{C}_{\Delta;\Gamma}$ then $[\delta ; \gamma] e \in \mathcal{E}^{C}_{A}$

Fundamental Lemma

If $\Delta ; \Gamma \vdash e : A$ and $[\delta ; \gamma] \in V^{C}_{\Delta;\Gamma}$ then $[\delta;\gamma] e \in \mathcal{E}^{C}_{A}$

Case $\dfrac{x : A \in \Gamma}{\Delta ; \Gamma \vdash x : A}$

1. $[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$ — Assumption
2. $[\delta;\gamma](x) = \gamma(x)$ — Def. of $[\delta;\gamma]e$
3. $\gamma(x) \in V^{C}_{A}$ — Def of $V^{C}_{\Delta;\Gamma}$
4. $\gamma(x) \in \mathcal{E}^{C}_{A}$ — Since $V^{C}_{A} \subseteq \mathcal{E}^{C}_{A}$

Fundamental Lemma

If $\Delta ; \Gamma \vdash e : A$ and $[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$ then $[\delta;\gamma] e \in \mathcal{E}^{C}_{A}$

Case $$\frac{x : A \in \Delta}{\Delta ; \Gamma \vdash \underline{x} : A}$$

1. $[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$ Assumption
2. $[\delta;\gamma](\underline{x}) = \delta(x)$ Def. of $[\delta;\gamma]e$
3. $\delta(x) \in V^{\emptyset}_{A}$ Def of $V^{C}_{\Delta;\Gamma}$
4. $\delta(x) \in V^{C}_{A}$ Kripke Monotonicity
5. $\delta(x) \in \mathcal{E}^{C}_{A}$ Since $V^{C}_{A} \subseteq \mathcal{E}^{C}_{A}$

Fundamental Lemma

Case $$\frac{\Delta; \Gamma \vdash e : \mathrm{Chan} \qquad \Delta; \Gamma \vdash e' : \mathbb{N}}{\Delta; \Gamma \vdash \mathrm{print}(e, e') : 1}$$

1. $[\delta; \gamma] \in V^{C}_{\Delta;\Gamma}$ — Assumption
2. $\Delta; \Gamma \vdash e : \mathrm{Chan}$ — Subderivation
3. $\Delta; \Gamma \vdash e' : \mathbb{N}$ — Subderivation

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

1. $[\delta;\gamma] \in \mathcal{V}^{C}_{\Delta;\Gamma}$ — Assumption
2. $\Delta;\Gamma \vdash e : \mathrm{Chan}$ — Subderivation
3. $\Delta;\Gamma \vdash e' : \mathbb{N}$ — Subderivation
4. $[\delta;\gamma]e \in \mathcal{E}^{C}_{\mathrm{Chan}}$ — Induction

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

1.	$[\delta;\gamma] \in \mathcal{V}^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta;\Gamma \vdash e : \mathrm{Chan}$	Subderivation
3.	$\Delta;\Gamma \vdash e' : \mathbb{N}$	Subderivation
4.	$[\delta;\gamma]e \in \mathcal{E}^{C}_{\mathrm{chan}}$	Induction
5.	$[\delta;\gamma]e' \in \mathcal{E}^{C}_{\mathbb{N}}$	Induction

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \text{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \text{print}(e,e') : 1}$

1. $[\delta;\gamma] \in V^C_{\Delta;\Gamma}$ — Assumption
2. $\Delta;\Gamma \vdash e : \text{Chan}$ — Subderivation
3. $\Delta;\Gamma \vdash e' : \mathbb{N}$ — Subderivation
4. $[\delta;\gamma]e \in \mathcal{E}^C_{\text{chan}}$ — Induction
5. $[\delta;\gamma]e' \in \mathcal{E}^C_{\mathbb{N}}$ — Induction
6. $[\delta;e]e \xrightarrow{w}{}^* v$, $\text{chans}(w) \subseteq C$, $v \in V^C_{\text{chan}}$ — Def. of $\mathcal{E}^C_{\text{chan}}$

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

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3.	$\Delta;\Gamma \vdash e' : \mathbb{N}$	Subderivation
4.	$[\delta;\gamma]e \in \mathcal{E}^{C}_{\mathrm{chan}}$	Induction
5.	$[\delta;\gamma]e' \in \mathcal{E}^{C}_{\mathbb{N}}$	Induction
6.	$[\delta;e]e \xrightarrow{w}{}^{*} v$, $\mathrm{chans}(w) \subseteq C$, $v \in V^{C}_{\mathrm{chan}}$	Def. of $\mathcal{E}^{C}_{\mathrm{chan}}$
7.	$v \in C$	Def of V^{C}_{chan}

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

1.	$[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta;\Gamma \vdash e : \mathrm{Chan}$	Subderivation
3.	$\Delta;\Gamma \vdash e' : \mathbb{N}$	Subderivation
4.	$[\delta;\gamma]e \in \mathcal{E}^{C}_{\mathrm{chan}}$	Induction
5.	$[\delta;\gamma]e' \in \mathcal{E}^{C}_{\mathbb{N}}$	Induction
6.	$[\delta;e]e \xrightarrow{\omega}{}^{*} v$, $\mathrm{chans}(\omega) \subseteq C$, $v \in V^{C}_{\mathrm{chan}}$	Def. of $\mathcal{E}^{C}_{\mathrm{chan}}$
7.	$v \in C$	Def of V^{C}_{chan}
8.	$[\delta;e]e' \xrightarrow{\omega'} v'$, $\mathrm{chans}(\omega') \subseteq C$, $v' \in V^{C}_{\mathbb{N}}$	Def. of $\mathcal{E}^{C}_{\mathbb{N}}$

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

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2.	$\Delta;\Gamma \vdash e : \mathrm{Chan}$	Subderivation
3.	$\Delta;\Gamma \vdash e' : \mathbb{N}$	Subderivation
4.	$[\delta;\gamma]e \in \mathcal{E}^{C}_{\mathrm{Chan}}$	Induction
5.	$[\delta;\gamma]e' \in \mathcal{E}^{C}_{\mathbb{N}}$	Induction
6.	$[\delta;e]e \xrightarrow{w}{}^{*} v$, $\mathrm{chans}(w) \subseteq C$, $v \in \mathcal{V}^{C}_{\mathrm{Chan}}$	Def. of $\mathcal{E}^{C}_{\mathrm{Chan}}$
7.	$v \in C$	Def of $\mathcal{V}^{C}_{\mathrm{Chan}}$
8.	$[\delta;e]e' \xrightarrow{w'} v'$, $\mathrm{chans}(w') \subseteq C$, $v' \in \mathcal{V}^{C}_{\mathbb{N}}$	Def. of $\mathcal{E}^{C}_{\mathbb{N}}$
9.	$v' \in \mathcal{V}^{C}_{\mathbb{N}}$	Def of $\mathcal{V}^{C}_{\mathbb{N}}$

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

1.	$[\delta;\gamma] \in V^C_{\Delta;\Gamma}$	Assumption
2.	$\Delta;\Gamma \vdash e : \mathrm{Chan}$	Subderivation
3.	$\Delta;\Gamma \vdash e' : \mathbb{N}$	Subderivation
4.	$[\delta;\gamma]e \in \mathcal{E}^C_{\mathrm{chan}}$	Induction
5.	$[\delta;\gamma]e' \in \mathcal{E}^C_{\mathbb{N}}$	Induction
6.	$[\delta;e]e \xrightarrow{\omega}{}^* v,\ \mathrm{chans}(\omega) \subseteq C,\ v \in V^C_{\mathrm{chan}}$	Def. of $\mathcal{E}^C_{\mathrm{chan}}$
7.	$v \in C$	Def of V^C_{chan}
8.	$[\delta;e]e' \xrightarrow{\omega'} v',\ \mathrm{chans}(\omega') \subseteq C,\ v' \in V^C_{\mathbb{N}}$	Def. of $\mathcal{E}^C_{\mathbb{N}}$
9.	$v \in V^C_{\mathbb{N}}$	Def of $V^C_{\mathbb{N}}$
10.	$\mathrm{print}(v,v') \xrightarrow{\mathrm{wr}(v,v')} ()$	Rule

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

1.	$[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta;\Gamma \vdash e : \mathrm{Chan}$	Subderivation
3.	$\Delta;\Gamma \vdash e' : \mathbb{N}$	Subderivation
4.	$[\delta;\gamma]e \in \mathcal{E}^{C}_{\mathrm{chan}}$	Induction
5.	$[\delta;\gamma]e' \in \mathcal{E}^{C}_{\mathbb{N}}$	Induction
6.	$[\delta;e]e \xrightarrow{w}{}^{*} v,\ \mathrm{chans}(w) \subseteq C,\ v \in V^{C}_{\mathrm{chan}}$	Def. of $\mathcal{E}^{C}_{\mathrm{chan}}$
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8.	$[\delta;e]e' \xrightarrow{w'} v,\ \mathrm{chans}(w') \subseteq C,\ v \in V^{C}_{\mathbb{N}}$	Def. of $\mathcal{E}^{C}_{\mathbb{N}}$
9.	$v \in V^{C}_{\mathbb{N}}$	Def of $V^{C}_{\mathbb{N}}$
10.	$\mathrm{print}(v,v') \xrightarrow{wr(v,v')} ()$	Rule
11.	$\mathrm{print}([\delta;\gamma]e, [\delta;\gamma]e') \xrightarrow{w \cdot w' \cdot wr(v,v')}{}^{*} ()$	Transitivity of $\longrightarrow^{*}$

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

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6.	$[\delta;e]e \xrightarrow{w}{}^{*} v,\ \mathrm{chans}(w) \subseteq C,\ v \in V^{C}_{\mathrm{chan}}$	Def. of $\mathcal{E}^{C}_{\mathrm{chan}}$
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8.	$[\delta;e]e' \xrightarrow{w'} v,\ \mathrm{chans}(w') \subseteq C,\ v \in V^{C}_{\mathbb{N}}$	Def. of $\mathcal{E}^{C}_{\mathbb{N}}$
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11.	$\mathrm{print}([\delta;\gamma]e, [\delta;\gamma]e') \xrightarrow{w \cdot w' \cdot wr(v,v')}{}^{*} ()$	Transitivity of $\longrightarrow^{*}$
12.	$\mathrm{chans}(w \cdot w' \cdot wr(v,v')) \subseteq C$	Immediate

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

1.	$[\delta;\gamma] \in V^C_{\Delta;\Gamma}$	Assumption
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3.	$\Delta;\Gamma \vdash e' : \mathbb{N}$	Subderivation
4.	$[\delta;\gamma]e \in \mathcal{E}^C_{\mathrm{chan}}$	Induction
5.	$[\delta;\gamma]e' \in \mathcal{E}^C_{\mathbb{N}}$	Induction
6.	$[\delta;e]e \xrightarrow{\omega}{}^* v,\ \mathrm{chans}(\omega) \subseteq C,\ v \in V^C_{\mathrm{chan}}$	Def. of $\mathcal{E}^C_{\mathrm{chan}}$
7.	$v \in C$	Def of V^C_{chan}
8.	$[\delta;e]e' \xrightarrow{\omega'} v,\ \mathrm{chans}(\omega') \subseteq C,\ v \in V^C_{\mathbb{N}}$	Def. of $\mathcal{E}^C_{\mathbb{N}}$
9.	$v \in V^C_{\mathbb{N}}$	Def of $V^C_{\mathbb{N}}$
10.	$\mathrm{print}(v,v') \xrightarrow{wr(v,v')} ()$	Rule
11.	$\mathrm{print}([\delta;\gamma]e, [\delta;\gamma]e') \xrightarrow{\omega \cdot \omega' \cdot wr(v,v')}{}^* ()$	Transitivity of $\longrightarrow^*$
12.	$\mathrm{chans}(\omega \cdot \omega' \cdot wr(v,v')) \subseteq C$	Immediate
13.	$() \in V^C_1$	Def. of V^C_1

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \mathrm{Chan} \qquad \Delta;\Gamma \vdash e' : \mathbb{N}}{\Delta;\Gamma \vdash \mathrm{print}(e,e') : 1}$

1.	$[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
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5.	$[\delta;\gamma]e' \in \mathcal{E}^{C}_{\mathbb{N}}$	Induction
6.	$[\delta;e]e \xrightarrow{w}{}^{*} v,\ \mathrm{chans}(w) \subseteq C,\ v \in V^{C}_{\mathrm{Chan}}$	Def. of $\mathcal{E}^{C}_{\mathrm{Chan}}$
7.	$v \in C$	Def of V^{C}_{Chan}
8.	$[\delta;e]e' \xrightarrow{w'} v,\ \mathrm{chans}(w') \subseteq C,\ v \in V^{C}_{\mathbb{N}}$	Def. of $\mathcal{E}^{C}_{\mathbb{N}}$
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11.	$\mathrm{print}([\delta;\gamma]e, [\delta;\gamma]e') \xrightarrow{w \cdot w' \cdot wr(v,v')}{}^{*} ()$	Transitivity of $\longrightarrow^{*}$
12.	$\mathrm{chans}(w \cdot w' \cdot wr(v,v')) \subseteq C$	Immediate
13.	$() \in V^{C}_{1}$	Def. of V^{C}_{1}
14.	$\mathrm{print}([\delta;\gamma]e, [\delta;\gamma]e') \in \mathcal{E}^{C}_{1}$	Def of $\mathcal{E}^{C}_{1}$

Fundamental Lemma

Case $\dfrac{\Delta; \cdot \vdash e : A}{\Delta; \Gamma \vdash \mathrm{box}(e) : \Box A}$

1. $[\delta;\gamma] \in V^{c}_{\Delta;\Gamma}$ Assumption
2. $\Delta; \cdot \vdash e : A$ Subderivation

Fundamental Lemma

Case $$\frac{\Delta ; \cdot \vdash e : A}{\Delta ; \Gamma \vdash \mathrm{box}(e) : \Box A}$$

1. $[\delta ; \gamma] \in V^{c}_{\Delta;\Gamma}$ — Assumption
2. $\Delta ; \cdot \vdash e : A$ — Subderivation
3. $[\delta ; \cdot] \in V^{\emptyset}_{\Delta;\cdot}$ — Def. of $V^{c}_{\Delta;\Gamma}$

Fundamental Lemma

Case $\dfrac{\Delta; \cdot \vdash e : A}{\Delta; \Gamma \vdash \mathrm{box}(e) : \Box A}$

1.	$[\delta; \gamma] \in V^{c}_{\Delta;\Gamma}$	Assumption
2.	$\Delta; \cdot \vdash e : A$	Subderivation
3.	$[\delta; \cdot] \in V^{\emptyset}_{\Delta;\cdot}$	Def. of $V^{c}_{\Delta;\Gamma}$
4.	$[\delta; \cdot]e \in \mathcal{E}^{\emptyset}_{A}$	Induction

Fundamental Lemma

Case $\dfrac{\Delta; \cdot \vdash e : A}{\Delta; \Gamma \vdash \mathrm{box}(e) : \Box A}$

1. $[\delta; \gamma] \in V^{c}_{\Delta;\Gamma}$ — Assumption
2. $\Delta; \cdot \vdash e : A$ — Subderivation
3. $[\delta; \cdot] \in V^{\phi}_{\Delta;\cdot}$ — Def. of $V^{c}_{\Delta;\Gamma}$
4. $[\delta; \cdot]e \in \mathcal{E}^{\phi}_{A}$ — Induction
5. $\mathrm{box}([\delta; \cdot]e) \xrightarrow{w}{}^{*} v$, $\mathrm{chans}(w) \subseteq \phi$, $v \in V^{\phi}_{A}$ — Def. of $\underline{\mathcal{E}}^{c}_{A}$

Fundamental Lemma

Case $\dfrac{\Delta; \cdot \vdash e : A}{\Delta; \Gamma \vdash \mathrm{box}(e) : \Box A}$

1.	$[\delta;\gamma] \in V^{c}_{\Delta;\Gamma}$	Assumption
2.	$\Delta; \cdot \vdash e : A$	Subderivation
3.	$[\delta; \cdot] \in V^{\phi}_{\Delta;\cdot}$	Def. of $V^{c}_{\Delta;\Gamma}$
4.	$[\delta;\cdot]e \in \mathcal{E}^{\phi}_{A}$	Induction
5.	$\mathrm{box}([\delta;\cdot]e) \xrightarrow{\omega}{}^{*} v$, $\mathrm{chans}(\omega) \subseteq \phi$, $v \in V^{\phi}_{A}$	Def. of $\mathcal{E}^{c}_{A}$
6.	$[\delta;\gamma]\,\mathrm{box}(e) \xrightarrow{\omega}{}^{*} v$	Def. of $[\delta;\gamma]e$

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Fundamental Lemma

Case $\dfrac{\Delta; \cdot \vdash e : A}{\Delta; \Gamma \vdash \mathrm{box}(e) : \Box A}$

1.	$[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta; \cdot \vdash e : A$	Subderivation
3.	$[\delta; \cdot] \in V^{\phi}_{\Delta;\cdot}$	Def. of $V^{C}_{\Delta;\Gamma}$
4.	$[\delta;\cdot]e \in \mathcal{E}^{\phi}_{A}$	Induction
5.	$\mathrm{box}([\delta;\cdot]e) \xrightarrow{w}{}^{*} v$, $\mathrm{chans}(w) \subseteq \phi$, $v \in V^{\phi}_{A}$	Def. of $\mathcal{E}^{C}_{A}$
6.	$[\delta;\gamma]\,\mathrm{box}(e) \xrightarrow{w}{}^{*} v$	Def. of $[\delta;\gamma]e$
7.	$\mathrm{chans}(w) \subseteq C$	Since $\phi \subseteq C$

Fundamental Lemma

Case $\dfrac{\Delta ; \cdot \vdash e : A}{\Delta ; \Gamma \vdash \mathrm{box}(e) : \Box A}$

1.	$[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta ; \cdot \vdash e : A$	Subderivation
3.	$[\delta; \cdot] \in V^{\phi}_{\Delta;\cdot}$	Def. of $V^{C}_{\Delta;\Gamma}$
4.	$[\delta;\cdot]e \in \mathcal{E}^{\phi}_{A}$	Induction
5.	$\mathrm{box}([\delta;\cdot]e) \xrightarrow{w}{}^{*} v$, $\mathrm{chans}(w) \subseteq \phi$, $v \in V^{\phi}_{A}$	Def. of $\mathcal{E}^{C}_{A}$
6.	$[\delta;\gamma]\,\mathrm{box}(e) \xrightarrow{w}{}^{*} v$	Def. of $[\delta;\gamma]e$
7.	$\mathrm{chans}(w) \subseteq C$	Since $\phi \subseteq C$
8.	$v \in V^{\phi}_{A}$	Kripke monotonicity

Fundamental Lemma

Case $\dfrac{\Delta ; \cdot \vdash e : A}{\Delta ; \Gamma \vdash \mathrm{box}(e) : \Box A}$

1.	$[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta ; \cdot \vdash e : A$	Subderivation
3.	$[\delta ; \cdot] \in V^{\phi}_{\Delta;\cdot}$	Def. of $V^{C}_{\Delta;\Gamma}$
4.	$[\delta;\cdot]e \in \mathcal{E}^{\phi}_{A}$	Induction
5.	$\mathrm{box}([\delta;\cdot]e) \xrightarrow{w}{}^{*} v$, $\mathrm{chans}(w) \subseteq \phi$, $v \in V^{\phi}_{A}$	Def. of $\mathcal{E}^{C}_{A}$
6.	$[\delta;\gamma]\,\mathrm{box}(e) \xrightarrow{w}{}^{*} v$	Def. of $[\delta;\gamma]e$
7.	$\mathrm{chans}(w) \subseteq C$	Since $\phi \subseteq C$
8.	$v \in V^{\phi}_{A} \Rightarrow \mathrm{box}(v) \in V^{C}_{\Box A}$	Kripke monotonicity
9.	$[\delta;\gamma]\,\mathrm{box}(e) \in \mathcal{E}^{C}_{A}$	Def of $\mathcal{E}^{C}_{A}$

Fundamental Lemma

Case $$\frac{\Delta ; \Gamma \vdash e : \Box A \qquad \Delta, x : A ; \Gamma \vdash e' : B}{\Delta ; \Gamma \vdash \text{let box}(x) = e \text{ in } e' : B}$$

1. $[\delta ; \gamma] \in V^{c}_{\Delta;\Gamma}$ — Assumption
2. $\Delta ; \Gamma \vdash e : \Box A$ — Subderivation
3. $\Delta, x : A ; \Gamma \vdash e' : B$ — Subderivation

Fundamental Lemma

Case $$\frac{\Delta; \Gamma \vdash e : \Box A \qquad \Delta, x : A; \Gamma \vdash e' : B}{\Delta; \Gamma \vdash \mathsf{let}\ \mathsf{box}(x) = e\ \mathsf{in}\ e' : B}$$

1. $[\delta; \gamma] \in \mathcal{V}^{c}_{\Delta; \Gamma}$ — Assumption
2. $\Delta; \Gamma \vdash e : \Box A$ — Subderivation
3. $\Delta, x : A; \Gamma \vdash e' : B$ — Subderivation
4. $[\delta; \gamma] e \in \mathcal{E}^{c}_{\Box A}$ — Induction

Fundamental Lemma

Case $\dfrac{\Delta ; \Gamma \vdash e : \Box A \qquad \Delta, x : A ; \Gamma \vdash e' : B}{\Delta ; \Gamma \vdash \mathrm{let\ box}(x) = e \ \mathrm{in}\ e' : B}$

1.	$[\delta;\gamma] \in V^c_{\Delta;\Gamma}$	Assumption
2.	$\Delta;\Gamma \vdash e : \Box A$	Subderivation
3.	$\Delta, x : A ; \Gamma \vdash e' : B$	Subderivation
4.	$[\delta;\gamma] e \in \mathcal{E}^c_{\Box A}$	Induction
5.	$[\delta;\gamma] e \xrightarrow{w}{}^{*} \hat{v}$	Def of $\mathcal{E}^c_{\Box A}$

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \Box A \qquad \Delta, x : A; \Gamma \vdash e' : B}{\Delta;\Gamma \vdash \text{let box}(x) = e \text{ in } e' : B}$

1. $[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$ Assumption
2. $\Delta;\Gamma \vdash e : \Box A$ Subderivation
3. $\Delta, x : A; \Gamma \vdash e' : B$ Subderivation
4. $[\delta;\gamma]e \in \mathcal{E}^{C}_{\Box A}$ Induction
5. $[\delta;\gamma]e \xrightarrow{\omega}{}^{*} \hat{v}$ Def of $\mathcal{E}^{C}_{\Box A}$
6. $\text{chans}(\omega) \subseteq C$

Fundamental Lemma

Case $$\frac{\Delta ; \Gamma \vdash e : \Box A \qquad \Delta, x : A ; \Gamma \vdash e' : B}{\Delta ; \Gamma \vdash \text{let box}(x) = e \text{ in } e' : B}$$

1.	$[\delta ; \gamma] \in V^{C}_{\Delta ; \Gamma}$	Assumption
2.	$\Delta ; \Gamma \vdash e : \Box A$	Subderivation
3.	$\Delta, x : A ; \Gamma \vdash e' : B$	Subderivation
4.	$[\delta ; \gamma] e \in \mathcal{E}^{C}_{\Box A}$	Induction
5.	$[\delta ; \gamma] e \xrightarrow{\omega}{}^{*} \hat{v}$	Def of $\mathcal{E}^{C}_{\Box A}$
6.	$\text{chans}(\omega) \subseteq C$	
7.	$\hat{v} \in V^{C}_{\Box A}$	Def of $V^{C}_{\Box A}$

Fundamental Lemma

Case $\dfrac{\Delta ; \Gamma \vdash e : \Box A \qquad \Delta, x : A ; \Gamma \vdash e' : B}{\Delta ; \Gamma \vdash \text{let box}(x) = e \text{ in } e' : B}$

1.	$[\delta ; \gamma] \in V^{C}_{\Delta ; \Gamma}$	Assumption
2.	$\Delta ; \Gamma \vdash e : \Box A$	Subderivation
3.	$\Delta, x : A ; \Gamma \vdash e' : B$	Subderivation
4.	$[\delta ; \gamma] e \in \mathcal{E}^{C}_{\Box A}$	Induction
5.	$[\delta ; \gamma] e \xrightarrow{\omega}{}^{*} \hat{v}$	Def of $\mathcal{E}^{C}_{\Box A}$
6.	$\text{chans}(\omega) \subseteq C$	
7.	$\hat{v} \in V^{C}_{\Box A}$	Def of $V^{C}_{\Box A}$
8.	$\hat{v} = \text{box}(v)$, $v \in V^{\emptyset}_{A}$	

Fundamental Lemma

Case $$\frac{\Delta;\Gamma \vdash e : \Box A \qquad \Delta, x : A; \Gamma \vdash e' : B}{\Delta;\Gamma \vdash \text{let } box(x) = e \text{ in } e' : B}$$

1.	$[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta;\Gamma \vdash e : \Box A$	Subderivation
3.	$\Delta, x : A; \Gamma \vdash e' : B$	Subderivation
4.	$[\delta;\gamma] e \in \mathcal{E}^{C}_{\Box A}$	Induction
5.	$[\delta;\gamma] e \xrightarrow{\omega}{}^{*} \hat{v}$	Def of $\mathcal{E}^{C}_{\Box A}$
6.	$chans(\omega) \subseteq C$	
7.	$\hat{v} \in V^{C}_{\Box A}$	Def of $V^{C}_{\Box A}$
8.	$\hat{v} = box(v)$, $v \in V^{\emptyset}_{A}$	
9.	$[(\delta, v/x);\gamma] \in V^{C}_{\Delta, x:A;\Gamma}$	Def of $V^{C}_{\Delta;\Gamma}$

Fundamental Lemma

Case $\dfrac{\Delta ; \Gamma \vdash e : \Box A \qquad \Delta, x : A ; \Gamma \vdash e' : B}{\Delta ; \Gamma \vdash \text{let box}(x) = e \text{ in } e' : B}$

1.	$[\delta ; \gamma] \in V^{C}_{\Delta ; \Gamma}$	Assumption
2.	$\Delta ; \Gamma \vdash e : \Box A$	Subderivation
3.	$\Delta, x : A ; \Gamma \vdash e' : B$	Subderivation
4.	$[\delta ; \gamma] e \in \mathcal{E}^{C}_{\Box A}$	Induction
5.	$[\delta ; \gamma] e \xrightarrow{\omega}{}^{*} \hat{v}$	Def of $\mathcal{E}^{C}_{\Box A}$
6.	$\text{chans}(\omega) \subseteq C$	
7.	$\hat{v} \in V^{C}_{\Box A}$	Def of $V^{C}_{\Box A}$
8.	$\hat{v} = \text{box}(v)$, $v \in V^{\emptyset}_{A}$	
9.	$[(\delta, v/x) ; \gamma] \in V^{C}_{\Delta, x : A ; \Gamma}$	Def of $V^{C}_{\Delta ; \Gamma}$
10.	$[(\delta, v/x) ; \gamma] e' \in \mathcal{E}^{C}_{B}$	Induction

Fundamental Lemma

Case $\dfrac{\Delta ; \Gamma \vdash e : \Box A \qquad \Delta, x : A ; \Gamma \vdash e' : B}{\Delta ; \Gamma \vdash \text{let box}(x) = e \text{ in } e' : B}$

1.	$[\delta ; \gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta ; \Gamma \vdash e : \Box A$	Subderivation
3.	$\Delta, x : A ; \Gamma \vdash e' : B$	Subderivation
4.	$[\delta ; \gamma] e \in \mathcal{E}^{C}_{\Box A}$	Induction
5.	$[\delta ; \gamma] e \xrightarrow{\omega}{}^{*} \hat{v}$	Def of $\mathcal{E}^{C}_{\Box A}$
6.	$\text{chans}(\omega) \subseteq C$	
7.	$\hat{v} \in V^{C}_{\Box A}$	Def of $V^{C}_{\Box A}$
8.	$\hat{v} = \text{box}(v)\ ,\ v \in V^{\emptyset}_{A}$	
9.	$[(\delta, v/x) ; \gamma] \in V^{C}_{\Delta, x : A ; \Gamma}$	Def of $V^{C}_{\Delta;\Gamma}$
10.	$[(\delta, v/x) ; \gamma] e' \in \mathcal{E}^{C}_{B}$	Induction
11.	$\text{let box}(x) = [\delta;\gamma] e \text{ in } [\delta, (x/x) ; \gamma] e'$ $\xrightarrow{\omega}{}^{*} \text{let box}(x) = \text{box}(v) \text{ in } [(\delta, x/x) ; \gamma] e' \xrightarrow{\cdot} [(\delta, v/x) ; \gamma] e'$	By congruence rules

Fundamental Lemma

Case $$\frac{\Delta ; \Gamma \vdash e : \Box A \qquad \Delta, x : A ; \Gamma \vdash e' : B}{\Delta ; \Gamma \vdash \text{let box}(x) = e \text{ in } e' : B}$$

1.	$[\delta ; \gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta ; \Gamma \vdash e : \Box A$	Subderivation
3.	$\Delta, x : A ; \Gamma \vdash e' : B$	Subderivation
4.	$[\delta ; \gamma] e \in \mathcal{E}^{C}_{\Box A}$	Induction
5.	$[\delta ; \gamma] e \xrightarrow{\omega}{}^{*} \hat{v}$	Def of $\mathcal{E}^{C}_{\Box A}$
6.	$\text{chans}(\omega) \subseteq C$	
7.	$\hat{v} \in V^{C}_{\Box A}$	Def of $V^{C}_{\Box A}$
8.	$\hat{v} = \text{box}(v) \;,\; v \in V^{\emptyset}_{A}$	
9.	$[(\delta, v/x) ; \gamma] \in V^{C}_{\Delta, x : A ; \Gamma}$	Def of $V^{C}_{\Delta;\Gamma}$
10.	$[(\delta, v/x) ; \gamma] e' \in \mathcal{E}^{C}_{B}$	Induction
11.	$\text{let box}(x) = [\delta ; \gamma] e \text{ in } [\delta, (x/x) ; \gamma] e'$ $\xrightarrow{\omega}{}^{*} \text{let box}(x) = \text{box}(v) \text{ in } [(\delta, x/x) ; \gamma] e' \longrightarrow [(\delta, v/x) ; \gamma] e'$	By congruence rules
12.	$[\delta ; \gamma](\text{let box}(x) = e \text{ in } e') \xrightarrow{\omega} [(\delta, v/x) ; \gamma] e'$	Def of $[\delta ; \gamma] e$

Fundamental Lemma

Case $\dfrac{\Delta;\Gamma \vdash e : \Box A \qquad \Delta, x:A;\Gamma \vdash e' : B}{\Delta;\Gamma \vdash \text{let box}(x) = e \text{ in } e' : B}$

1.	$[\delta;\gamma] \in V^{C}_{\Delta;\Gamma}$	Assumption
2.	$\Delta;\Gamma \vdash e : \Box A$	Subderivation
3.	$\Delta, x: A;\Gamma \vdash e' : B$	Subderivation
4.	$[\delta;\gamma]e \in \mathcal{E}^{C}_{\Box A}$	Induction
5.	$[\delta;\gamma]e \xrightarrow{\omega}{}^{*} \hat{v}$	Def of $\mathcal{E}^{C}_{\Box A}$
6.	$\text{chans}(\omega) \subseteq C$	
7.	$\hat{v} \in V^{C}_{\Box A}$	Def of $V^{C}_{\Box A}$
8.	$\hat{v} = \text{box}(v)$, $v \in V^{\emptyset}_{A}$	
9.	$[(\delta, v/x);\gamma] \in V^{C}_{\Delta, x:A;\Gamma}$	Def of $V^{C}_{\Delta;\Gamma}$
10.	$[(\delta, v/x);\gamma]e' \in \mathcal{E}^{C}_{B}$	Induction
11.	$\text{let box}(x) = [\delta;\gamma]e \text{ in } [\delta,(x/x);\gamma]e'$ $\xrightarrow{\omega}{}^{*} \text{let box}(x) = \text{box}(v) \text{ in } [(\delta, x/x);\gamma]e' \longrightarrow [(\delta, v/x);\gamma]e'$	By congruence rules
12.	$[\delta;\gamma](\text{let box}(x) = e \text{ in } e') \xrightarrow{\omega} [(\delta, v/x);\gamma]e'$	Def of $[\delta;\gamma]e$
13.	$[\delta;\gamma](\text{let box}(x) = e \text{ in } e') \in \mathcal{E}^{C}_{B}$	C-Congruence

Fundamental Lemma

Case $\dfrac{\Delta; \Gamma, x:A \vdash e : B}{\Delta; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta; \Gamma, x:A \vdash e : B$ Subderivation
1. $[\delta; \gamma] \in V^{C}_{\Delta;\Gamma}$ Assumption

Fundamental Lemma

Case $\dfrac{\Delta; \Gamma, x:A \vdash e : B}{\Delta; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta; \Gamma, x:A \vdash e : B$ Subderivation
1. $[\delta; \gamma] \in V^C_{\Delta;\Gamma}$ Assumption

WTS $[\delta; \gamma](\lambda x. e) \in V^C_{A \to B}$

Fundamental Lemma

Case $\dfrac{\Delta; \Gamma, x:A \vdash e : B}{\Delta; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta; \Gamma, x:A \vdash e : B$ Subderivation
1. $[\delta; \gamma] \in V^{C}_{\Delta;\Gamma}$ Assumption

WTS $[\delta;\gamma](\lambda x. e) \in V^{C}_{A \to B}$

2. Assume $C' \geq C$, $v \in V^{C'}_{A}$

Fundamental Lemma

Case $\dfrac{\Delta; \Gamma, x:A \vdash e : B}{\Delta; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta; \Gamma, x:A \vdash e : B$ Subderivation
1. $[\delta; \gamma] \in V^{C}_{\Delta;\Gamma}$ Assumption

WTS $[\delta;\gamma](\lambda x. e) \in V^{C}_{A \to B}$

2. Assume $C' \geq C$, $v \in V^{C'}_{A}$
3. $[\delta;\gamma] \in V^{C'}_{\Delta;\Gamma}$ Kripke Monotonicity

Fundamental Lemma

Case $\dfrac{\Delta; \Gamma, x:A \vdash e : B}{\Delta; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta; \Gamma, x:A \vdash e : B$ Subderivation
1. $[\delta; \gamma] \in V^{C}_{\Delta;\Gamma}$ Assumption

WTS $[\delta;\gamma](\lambda x. e) \in V^{C}_{A \to B}$

2. Assume $C' \geq C$, $v \in V^{C'}_{A}$
3. $[\delta;\gamma] \in V^{C'}_{\Delta;\Gamma}$ Kripke Monotonicity
4. $(\lambda x. [\delta;(\gamma, x/x)] e)\ v \longrightarrow [v/x][\delta;\gamma, x/x] e$ Reduction

Fundamental Lemma

Case $\dfrac{\Delta ; \Gamma, x:A \vdash e : B}{\Delta ; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta ; \Gamma, x:A \vdash e : B$ — Subderivation
1. $[\delta ; \gamma] \in V^{C}_{\Delta;\Gamma}$ — Assumption

WTS $[\delta;\gamma](\lambda x. e) \in V^{C}_{A \to B}$

2. Assume $C' \geq C$, $v \in V^{C'}_{A}$
3. $[\delta;\gamma] \in V^{C'}_{\Delta;\Gamma}$ — Kripke Monotoniciy
4. $(\lambda x. [\delta;(\gamma, x/x)]e)\ v \longrightarrow [v/x][\delta;\gamma, x/x]\,e$ — Reduction
5. $[v/x][\delta;(\gamma, x/x)] = [\delta; (\gamma, v/x)]$ — Def. of subst.

Fundamental Lemma

Case $\dfrac{\Delta ; \Gamma, x : A \vdash e : B}{\Delta ; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta ; \Gamma, x : A \vdash e : B$ — Subderivation
1. $[\delta ; \gamma] \in V^{C}_{\Delta ; \Gamma}$ — Assumption

WTS $[\delta ; \gamma](\lambda x. e) \in V^{C}_{A \to B}$

2. Assume $C' \geq C$, $v \in V^{C'}_{A}$
3. $[\delta ; \gamma] \in V^{C'}_{\Delta ; \Gamma}$ — Kripke Monotoniciety
4. $(\lambda x. [\delta ; (\gamma, x/x)] e)\ v \longrightarrow [v/x][\delta ; \gamma, x/x]\, e$ — Reduction
5. $[v/x][\delta ; (\gamma, x/x)] = [\delta ; (\gamma, v/x)]$ — Def. of subst.
6. $[\delta ; (\gamma, v/x)] \in V^{C'}_{\Delta ; \Gamma, x : A}$ — Def of $V^{C}_{\Delta ; \Gamma}$

Fundamental Lemma

Case $\dfrac{\Delta; \Gamma, x:A \vdash e : B}{\Delta; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta; \Gamma, x:A \vdash e : B$ — Subderivation
1. $[\delta; \gamma] \in V^{C}_{\Delta;\Gamma}$ — Assumption

WTS $[\delta;\gamma](\lambda x. e) \in V^{C}_{A \to B}$

2. Assume $C' \geq C$, $v \in V^{C'}_{A}$
3. $[\delta;\gamma] \in V^{C'}_{\Delta;\Gamma}$ — Kripke Monotoniciity
4. $(\lambda x. [\delta;(\gamma, x/x)] e)\ v \longrightarrow [v/x][\delta;\gamma, x/x] e$ — Reduction
5. $[v/x][\delta;(\gamma, x/x)] = [\delta; (\gamma, v/x)]$ — Def. of subst.
6. $[\delta;(\gamma, v/x)] \in V^{C'}_{\Delta;\Gamma, x:A}$ — Def of $V^{C}_{\Delta;\Gamma}$
7. $[\delta;(\gamma, v/x)]e \in \mathcal{E}^{C'}_{B}$ — Induction

Fundamental Lemma

Case $\dfrac{\Delta ; \Gamma, x:A \vdash e : B}{\Delta ; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta ; \Gamma, x:A \vdash e : B$ — Subderivation
1. $[\delta ; \gamma] \in V^{C}_{\Delta ; \Gamma}$ — Assumption

WTS $[\delta ; \gamma](\lambda x. e) \in V^{C}_{A \to B}$

2. Assume $C' \geq C$, $v \in V^{C'}_{A}$
3. $[\delta ; \gamma] \in V^{C'}_{\Delta ; \Gamma}$ — Kripke Monotonicity
4. $(\lambda x. [\delta ; (\gamma, x/x)] e)\ v \longrightarrow [v/x][\delta ; \gamma, x/x] e$ — Reduction
5. $[v/x][\delta ; (\gamma, x/x)] = [\delta ; (\gamma, v/x)]$ — Def. of subst.
6. $[\delta ; (\gamma, v/x)] \in V^{C'}_{\Delta ; \Gamma, x:A}$ — Def of $V^{C}_{\Delta ; \Gamma}$
7. $[\delta ; (\gamma, v/x)] e \in \mathcal{E}^{C'}_{B}$ — Induction

8. $[\delta ; \gamma](\lambda x. e) \in V^{C}_{A \to B}$

Fundamental Lemma

Case $\dfrac{\Delta ; \Gamma, x:A \vdash e : B}{\Delta ; \Gamma \vdash \lambda x. e : A \to B}$

0. $\Delta ; \Gamma, x:A \vdash e : B$ Subderivation
1. $[\delta ; \gamma] \in V^{C}_{\Delta ; \Gamma}$ Assumption

WTS $[\delta ; \gamma](\lambda x. e) \in V^{C}_{A \to B}$

2. Assume $C' \geq C$, $v \in V^{C'}_{A}$
3. $[\delta ; \gamma] \in V^{C'}_{\Delta ; \Gamma}$ Kripke Monotoniciy
4. $(\lambda x. [\delta ; (\gamma, x/x)] e)\ v \longrightarrow [v/x][\delta ; \gamma, x/x] e$ Reduction
5. $[v/x][\delta ; (\gamma, x/x)] = [\delta ; (\gamma, v/x)]$ Def. of subst.
6. $[\delta ; (\gamma, v/x)] \in V^{C'}_{\Delta ; \Gamma, x:A}$ Def of $V^{C}_{\Delta ; \Gamma}$
7. $[\delta ; (\gamma, v/x)] e \in \mathcal{E}^{C'}_{B}$ Induction
8. $[\delta ; \gamma](\lambda x. e) \in V^{C}_{A \to B}$
9. $[\delta ; \gamma](\lambda x. e) \in \mathcal{E}^{C}_{A \to B}$ $V^{C}_{A \to B} \subseteq \mathcal{E}^{C}_{A \to B}$

Capability Safety

If $\cdot ; \cdot \vdash e : A$ then $e \xrightarrow{\cdot}{}^{*} v$

Proof: $[\emptyset ; \emptyset] \in V^{\emptyset}_{\cdot ; \cdot}$

$[\emptyset ; \emptyset] e \in \mathcal{E}^{\emptyset}_{A}$

$e \in \mathcal{E}^{\emptyset}_{A}$

$e \xrightarrow{\omega}{}^{*} v$, $v \in V^{\emptyset}_{A}$, $chan(\omega) \subseteq \emptyset$

Since $chan(\omega) \subseteq \emptyset$, $\omega = \cdot$

All channels must be passed in as arguments!