Quantified Satisfiability and Its Synthesis & Verification Applications
FLOLAC 2023

Problem Set

August 24-25, 2023

1 [QBF]

Prove or disprove the following implications:

(a) $\exists x, \exists y. f(x, y, z) \rightarrow \forall x, \forall y. f(x, y, z)$

(b) $\forall x, \forall y. f(x, y, z) \rightarrow \exists x, \exists y. f(x, y, z)$

(c) $\exists x, \forall y. f(x, y, z) \rightarrow \forall y, \exists x. f(x, y, z)$

(d) $\exists x, \forall y. f(x, y, z) \rightarrow \forall x, \exists y. f(x, y, z)$

2 [BDD]

Let $f = ab(c + d') + (ab' + a'b)(c'd + cd')$. Draw the ROBDD of f with variable ordering $a < b < c < d$ (with a on top).

3 [QBF Satisfiability]

Consider the QBF $\Phi = \forall a, \exists b, \forall c, \exists d. (a'b'd + a'bc + acd' + abc'd + b'cd)$.

(a) Determine the truth or falsity of Φ.

(b) Derive the Skolem or Herbrand functions of Φ.
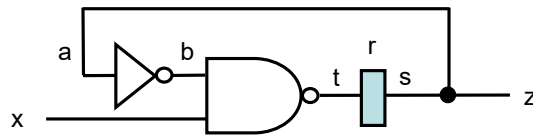
4 [DQBF Satisfiability]

Consider the DQBF $\Phi = \forall a, \forall c, \exists b(a), \exists d(c)). (a'b'd + a'bc + acd' + abc'd + b'cd)$, where b depends on a and d depends on (c).

(a) Determine the truth or falsity of Φ.

(b) Are there Skolem functions of Φ? If yes, derive them.

5 [CNF Conversion and Bounded Model Checking]

Consider the sequential circuit C in Figure 1, where register r is of initial value 1 at time $t = 0$.

(a) Give a CNF formula that checks whether the output z can have value 0 at time $t = 1$.

(b) Give a CNF formula that checks whether the output z can have value 0 at time $t = 2$.

Fig. 1. A sequential circuit.