Programming Language Theory

Imperative Language Constructs: Exercises

姓名： 學號： 系級:

1. (25%) Let T denote $\lambda f x.\, f(fx)$ and G denote $\lambda z.\,(z \times 2)$. Derive $(TG3), \sigma \Downarrow 12, \sigma$.

$$\frac{\dfrac{\dfrac{}{T, \sigma \Downarrow T, \sigma} \quad \dfrac{}{G, \sigma \Downarrow G, \sigma} \quad \dfrac{}{(\lambda x.\, G(Gx)), \sigma \Downarrow (\lambda x.\, G(Gx)), \sigma}}{TG, \sigma \Downarrow (\lambda x.\, G(Gx)), \sigma} \quad \dfrac{}{3, \sigma \Downarrow 3, \sigma} \quad D_{1,1}}{TG3, \sigma \Downarrow 12, \sigma}$$

(a) $D_{1,1}$:

$$\frac{\dfrac{}{G, \sigma \Downarrow G, \sigma} \quad \dfrac{\dfrac{}{G, \sigma \Downarrow G, \sigma} \quad \dfrac{}{3, \sigma \Downarrow 3, \sigma} \quad D_{1,2}}{G3, \sigma \Downarrow 6, \sigma} \quad D_{1,3}}{G(G3), \sigma \Downarrow 12, \sigma}$$

(b) $D_{1,2}$:

$$\frac{\dfrac{}{3, \sigma \Downarrow 3, \sigma} \quad \dfrac{}{2, \sigma \Downarrow 2, \sigma} \quad \sigma(\times, 3, 2) = 6}{3 \times 2, \sigma \Downarrow 6, \sigma}$$

(c) $D_{1,3}$:

$$\frac{\dfrac{}{6, \sigma \Downarrow 6, \sigma} \quad \dfrac{}{2, \sigma \Downarrow 2, \sigma} \quad \sigma(\times, 6, 2) = 12}{6 \times 2, \sigma \Downarrow 12, \sigma}$$

2. (15%) Write down the evaluation rule for let expressions.

$$\frac{e_1, \sigma \Downarrow v_1, \sigma_1 \qquad e_2[v_1/x], \sigma_1 \Downarrow v_2, \sigma_2}{(\text{let } x = e_1 \text{ in } e_2), \sigma \Downarrow v_2, \sigma_2}$$

3. (25%) Let n be any integer. Evaluate the following term with empty store.

$$(\text{let } r = \text{ref } n \text{ in } (r := !r + 1;\ !r)), \cdot \Downarrow \ ?\ ,\ ?$$

Let σ be $\cdot[l := n]$ and σ_1 be $\cdot[l := n+1]$ in the following derivations.

$$\frac{\dfrac{\dfrac{}{n, \cdot \Downarrow n, \cdot}}{\text{ref } n, \cdot \Downarrow l, \sigma} \qquad \dfrac{\dfrac{\dfrac{}{l, \sigma \Downarrow l, \sigma} \qquad D_{3,1} \qquad l \in \text{dom}(\sigma) \qquad \sigma_1 = \sigma[l := n+1]}{l := !l + 1, \sigma \Downarrow (), \sigma_1} \qquad D_{3,2}}{(l := !l + 1;\ !l), \sigma \Downarrow n+1, \sigma_1}}{(\text{let } r = \text{ref } n \text{ in } (r := !r + 1;\ !r)), \cdot \Downarrow n+1, \sigma_1}$$

- $D_{3,1}$:

$$\frac{\dfrac{\dfrac{}{l, \sigma \Downarrow l, \sigma}}{!l, \sigma \Downarrow n, \sigma} \qquad \dfrac{}{1, \sigma \Downarrow 1, \sigma} \qquad \delta(+, n, 1) = n+1}{!l + 1, \sigma \Downarrow n+1, \sigma}$$

- $D_{3,2}$:

$$\frac{\dfrac{}{l, \sigma_1 \Downarrow l, \sigma_1}}{!l, \sigma_1 \Downarrow n+1, \sigma_1}$$

4. (35%) Show that the following rules are both admissible. That is, for each of the rules, write down a segment of derivation with the same premises and conclusion using existing rules.

(20%)

$$\frac{e_1, \sigma \Downarrow \mathsf{true}, \sigma_1 \qquad e_2, \sigma_1 \Downarrow v, \sigma_2 \qquad (\mathsf{while}\ e_1\ \mathsf{do}\ e_2), \sigma_2 \Downarrow v', \sigma'}{(\mathsf{while}\ e_1\ \mathsf{do}\ e_2), \sigma \Downarrow v', \sigma'}$$

(15%)

$$\frac{e_1, \sigma \Downarrow \mathsf{false}, \sigma'}{(\mathsf{while}\ e_1\ \mathsf{do}\ e_2), \sigma \Downarrow (), \sigma'}$$

(a) (20%)

$$\dfrac{\dfrac{e_1, \sigma \Downarrow \mathsf{true}, \sigma_1 \qquad \dfrac{e_2, \sigma_1 \Downarrow v, \sigma_2 \qquad (\mathsf{while}\ e_1\ \mathsf{do}\ e_2), \sigma_2 \Downarrow v', \sigma'}{(e_2; (\mathsf{while}\ e_1\ \mathsf{do}\ e_2)), \sigma_1 \Downarrow v', \sigma'}}{\mathsf{if}\ e_1\ \mathsf{then}\ (e_2; \mathsf{while}\ e_1\ \mathsf{do}\ e_2)\ \mathsf{else}\ (), \sigma \Downarrow v', \sigma'}}{(\mathsf{while}\ e_1\ \mathsf{do}\ e_2), \sigma \Downarrow v', \sigma'}$$

(b) (15%)

$$\dfrac{\dfrac{e_1, \sigma \Downarrow \mathsf{false}, \sigma' \qquad \dfrac{}{(), \sigma' \Downarrow (), \sigma'}}{\mathsf{if}\ e_1\ \mathsf{then}\ (e_2; \mathsf{while}\ e_1\ \mathsf{do}\ e_2)\ \mathsf{else}\ (), \sigma \Downarrow v', \sigma'}}{(\mathsf{while}\ e_1\ \mathsf{do}\ e_2), \sigma \Downarrow v', \sigma'}$$