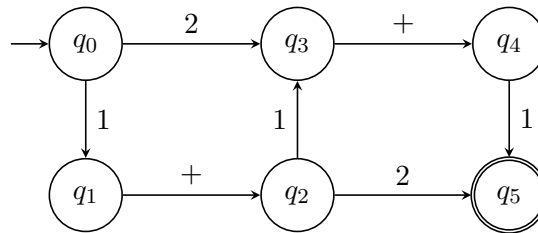


Suggested Solutions

[Compiled on September 4, 2017]

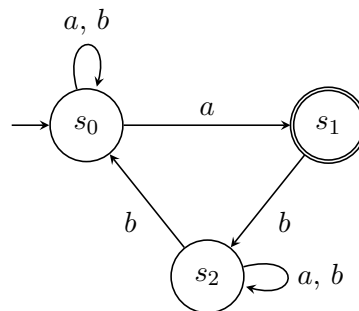
- Given an alphabet $\{1, 2, +\}$, draw a finite state automaton such that the automaton accepts words evaluated to 3.

Solution.

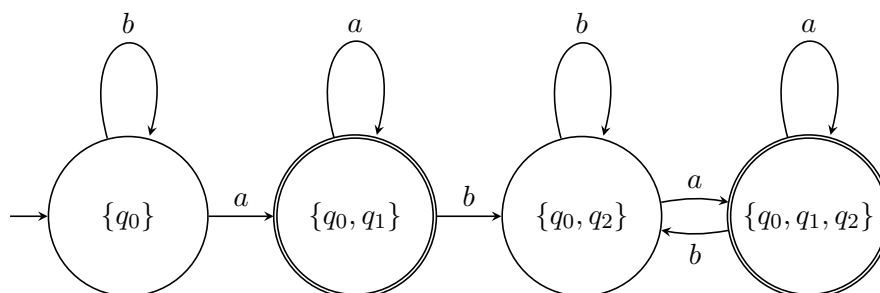


□

- Apply subset construction to determinize the following automaton.



Solution.



□

- Let $M_1 = (Q_1, \Sigma, \delta_1, I_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, I_2, F_2)$ be two NFAs. Construct an NFA M_3 such that $L(M_3) = L(M_1) \setminus L(M_2)$. Please describe the components of M_3 in detail.

Solution. Observe that $L(M_3) = L(M_1) \cap (\Sigma^* \setminus L(M_2))$. The automaton M_3 can be obtained by taking the intersection of M_1 and the complement of M_2 . The complement of M_2 can be obtained by subset construction followed by complementing accepting states. Define $M_3 = (Q_1 \times 2^{Q_2}, \Sigma, \Delta, I_1 \times \{I_2\}, G)$ where

- $(q', rs') \in \Delta((q, rs), a)$ for all $a \in \Sigma$ if and only if
 - $q' \in \delta_1(q, a)$, and
 - $rs' = \bigcup_{r \in rs} \delta_2(r, a)$, and
- $G = \{(q, rs) \mid q \in F_1, rs \subseteq Q_2, \text{ and } rs \cap F_2 = \emptyset\}$

□

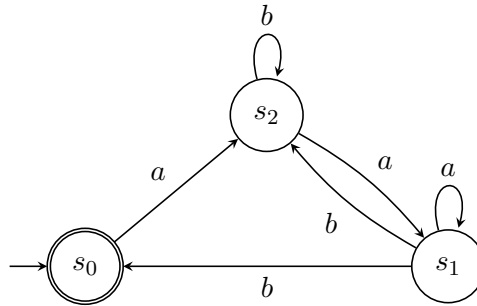
4. Write regular expressions to describe the following languages. ($\Sigma = \{a, b\}$)
- $\{w \mid \text{the length of } w \text{ is even}\}$
 - $\{w \mid w \text{ has at most two } b\text{'s}\}$
 - $\{w \mid \text{every } a \text{ in } w \text{ is followed by } b\}$

Solution.

- $(\Sigma\Sigma)^*$
- $(a^*) + (a^*ba^*) + (a^*ba^*ba^*)$
- $(b^*(ab)^*)^*$

□

5. Express the language of the following automaton by a regular expression.



Solution. Define the following equation system.

$$Q_0 = aQ_2 + \epsilon \tag{1}$$

$$Q_1 = aQ_1 + bQ_0 + bQ_2 \tag{2}$$

$$Q_2 = aQ_1 + bQ_2 \tag{3}$$

By equations 2 and 1, we have

$$\begin{aligned}
 Q_1 &= aQ_1 + bQ_0 + bQ_2 \\
 &= aQ_1 + b(aQ_2 + \epsilon) + bQ_2 \\
 &= aQ_1 + bQ_2 + b.
 \end{aligned}$$

By Ardens Lemma,

$$Q_1 = a^*(bQ_2 + b). \tag{4}$$

By equations 3 and 4, we have

$$\begin{aligned} Q_2 &= aQ_1 + bQ_2 \\ &= a(a^*(bQ_2 + b)) + bQ_2 \\ &= (aa^*b + b)Q_2 + aa^*b. \end{aligned}$$

By Ardens Lemma,

$$Q_2 = (aa^*b + b)^*(aa^*b). \quad (5)$$

Finally by equations 1 and 5, we have

$$\begin{aligned} Q_0 &= aQ_2 + \epsilon \\ &= a(aa^*b + b)^*(aa^*b) + \epsilon. \end{aligned}$$

Thus, the language of the automaton can be expressed in the regular expression $a(aa^*b + b)^*(aa^*b) + \epsilon$. \square

6. Write WS1S formulas to describe the following words.

- (a) Only a 's can occur between any two occurrences of b 's
- (b) Has an odd length (please start with \exists)

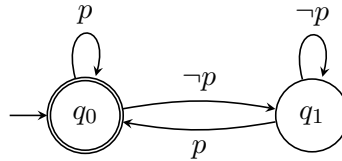
Solution.

- (a) $\forall x.\forall y.((P_b(x) \wedge P_b(y) \wedge x < y) \rightarrow (\forall z.(x < z \wedge z < y) \rightarrow P_a(z)))$
- (b) $\exists f.\exists l.\exists X.(first(f) \wedge last(l) \wedge X(f) \wedge X(l) \wedge \forall y.\forall z.(S(y, z) \rightarrow (X(y) \leftrightarrow \neg X(z))))$

\square

7. Draw a Büchi automaton that accepts infinite words where p holds infinitely many times. ($\Sigma = \{p, \neg p\}$)

Solution.



\square

8. Express the following sentences in LTL formulas.

- (a) “ p occurs infinitely often”
- (b) “whenever a message is sent, eventually an acknowledgement will be received”

Solution.

- (a) $\mathbf{G F p}$
- (b) $\mathbf{G}(sent \rightarrow \mathbf{F ack})$

\square