Exercises #2

[Compiled on August 31, 2017]

1. Let *max* be a function that returns the maximal number between two input numbers. Write a specification of *max* as precise as possible.

   - $\{?\}max(x, y)\{?\}$

2. Write the specification of a function that concatenates two integer lists. You may define other functions of list and use them in the specification.

   - List of integers is defined as $list ::= nil \mid cons(Int, list)$.

3. Complete the proof outline.

   $\{x \geq 0 \wedge y \geq 0 \wedge gcd(x, y) = gcd(m, n)\}$
   while $x \neq 0 \wedge y \neq 0$ **do**
   if $x < y$ **then**
   x, y := y, x
   fi;
   x := x − y
   od
   $\{(x = 0 \wedge y \geq 0 \wedge y = gcd(x, y) = gcd(m, n)) \vee$
   $(x \geq 0 \wedge y = 0 \wedge x = gcd(x, y) = gcd(m, n))\}$

4. Conside the following program.

```
x = nil;
i = 0;
while(i < n) {
  x = cons(i, x);
  i = i + 1;
}
j = 0
while(j < n) {
  assert(x != nil)
  x = del(x);
  j = j + 1;
}
```

   Assume $n \geq 0$ and

   - $list(0, x, x)$ for all x
   - $list(0, x, z) \rightarrow x = z$
   - $x = cons(a, b) \wedge list(n, b, z) \leftrightarrow list(n + 1, x, z)$
   - $list(n, x, z) \wedge y = del(x) \wedge n > 0 \rightarrow list(n - 1, y, z)$
   - $list(n, x, z) \wedge n > 0 \rightarrow x \neq nil$

Either show that the assertion wont be violated or find a counterexample that violates the assertion. ($list(n, x, y)$: x points to a list ended at y with length n.)