

Exercises #2

[Compiled on August 31, 2017]

1. Let max be a function that returns the maximal number between two input numbers. Write a specification of max as precise as possible.

- $\{?\}max(x, y)\{?\}$

2. Write the specification of a function that concatenates two integer lists. You may define other functions of list and use them in the specification.

- List of integers is defined as $list ::= nil \mid cons(Int, list)$.

3. Complete the proof outline.

```

 $\{x \geq 0 \wedge y \geq 0 \wedge gcd(x, y) = gcd(m, n)\}$ 
while  $x \neq 0 \wedge y \neq 0$  do
  if  $x < y$  then
     $x, y := y, x$ 
  fi;
   $x := x - y$ 
od
 $\{(x = 0 \wedge y \geq 0 \wedge y = gcd(x, y) = gcd(m, n)) \vee$ 
 $(x \geq 0 \wedge y = 0 \wedge x = gcd(x, y) = gcd(m, n))\}$ 

```

4. Consider the following program.

```

 $x = nil$ ;
 $i = 0$ ;
while( $i < n$ ) {
   $x = cons(i, x)$ ;
   $i = i + 1$ ;
}
 $j = 0$ 
while( $j < n$ ) {
  assert( $x \neq nil$ )
   $x = del(x)$ ;
   $j = j + 1$ ;
}

```

Assume $n \geq 0$ and

- $list(0, x, x)$ for all x
- $list(0, x, z) \rightarrow x = z$
- $x = cons(a, list(n, b, z)) \leftrightarrow list(n + 1, x, z)$
- $list(n, x, z) \wedge y = del(x) \wedge n > 0 \rightarrow list(n - 1, y, z)$

Either show that the assertion won't be violated or find a counterexample that violates the assertion.