

Exercises

[Compiled on August 30, 2017]

1. Use the semantic method to argue the validity of the following Σ_E -formulae, or identify a counterexample (a falsifying T_E -interpretation).

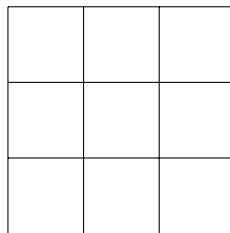
(a) $f(x, y) = f(y, x) \rightarrow f(a, y) = f(y, a)$

(b) $f(g(x)) = g(f(x)) \wedge f(g(f(y))) = x \wedge f(y) = x \rightarrow g(f(x)) = x$

2. Given the following 3×3 grid, we would like to find a way to fill the grid with numbers from 1 to 9 such that

- summations of every row, every column, and every diagonal are the same, and
- each number can appear only once.

Try to write an SMT formula such that the way exists if the SMT formula is satisfiable.



3. Apply the decision procedure for T_E to the following Σ_E -formulae. Provide a level of details as in slides.

(a) $f(x, y) = f(y, x) \wedge f(a, y) \neq f(y, a)$

(b) $f(g(x)) = g(f(x)) \wedge f(g(f(y))) = x \wedge f(y) = x \wedge g(f(x)) \neq x$

(c) $f(f(f(a))) = f(f(a)) \wedge f(f(f(f(a)))) = a \wedge f(a) \neq a$

(d) $p(x) \wedge f(f(x)) = x \wedge f(f(f(x))) = x \wedge \neg p(f(x))$

4. Apply the decision procedure for T_{cons} to the following T_{cons} -formulae. Please write down the call sequence to the *MERGE* procedure, draw the final DAG, and draw the final DAG.

(a) $car(x) = y \wedge cdr(x) = z \wedge x \neq cons(y, z)$

(b) $\neg atom(x) \wedge car(x) = y \wedge cdr(x) = z \wedge x \neq cons(y, z)$

5. Apply the decision procedure for quantifier-free T_A to the following Σ_A -formulae.

(a) $a\langle i \triangleleft e \rangle[j] = e \wedge i \neq j$

(b) $a\langle i \triangleleft e \rangle\langle j \triangleleft f \rangle[k] = g \wedge j \neq k \wedge i = j \wedge a[k] \neq g$