

[Tutorial 1 (Propositional Logic)]

This tutorial consists of an assortment of exercises from the slides (Session 1) and an activity of solving a murder mystery puzzle. [Needless to say, you should attempt to do all the exercises from the slides.]

From the lecture slides

[40–45 mins]

First make sure that you are familiar with the truth tables for all the connectives (check online if you haven't!). Start by working through the following easy exercises. All the proofs can be done by simply writing down the truth tables.

1. Prove that $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$
2. Prove De Morgan's Laws
3. Prove distributivity of \wedge
4. Express the all-true formula \top in terms of \neg, \vee .
5. Express the all-false formula \perp in terms of \neg, \wedge .
6. Prove that \wedge is associative and commutative.
7. Prove Contraposition: $A \rightarrow B \equiv \neg B \rightarrow \neg A$
8. Prove Modus Ponens: $(A \rightarrow B) \wedge A \equiv A \wedge B$

Minimum set of logical connectives

The task now is to show that each of the following sets of connectives is sufficient for expressing all propositional formulas:

- \neg, \wedge
- \neg, \vee

Explicitly state when you use the substitution principle.

Validity/satisfiability

1. Argue that the following formula is valid.

$$((\text{Eat} \rightarrow \neg\text{Starve}) \wedge \text{Eat}) \rightarrow \neg\text{Starve}$$

2. Prove satisfiability, and disprove validity of:

$$((\text{Eat} \rightarrow \neg\text{Starve}) \wedge \neg\text{Starve}) \rightarrow \text{Eat}$$

[After you've completed the murder mystery riddle (next activity), at home you may try to formalise and prove validity of: "If Eric studies, he does not fail exams. If Eric does not play too often, he studies. Eric fails exams. Thus, Eric plays too often."]

Solving a murder mystery riddle with logic

[40–45 mins]

In this activity, we are going to apply *formal* logic to solve a murder mystery riddle.

Learning goals:

- Recognizing logic signals in natural language.
- Applying *formal* logic to systematically solve the puzzle.

A warm up exercise

[10–15 mins]

Let's first solve something boring, but will prepare you for the murder mystery riddle:

Let's play a game. Flip a coin. Heads, I win. Tails, you lose. (*)

Your task is to argue that I always win.

Step 1. Carefully formulate the stated facts in propositional logic. Firstly, you must carefully pick your propositions. Here are the possible propositions:

- H — head
- T — tail
- I — I win
- Y — you win

Task: Write a propositional formula F for (*)

Step 2. Clearly state some reasonable assumptions that arise from ambiguity of natural language. Here they are:

(A1) $H \oplus T$ (or equivalently, $H \rightarrow \neg T \wedge \neg H \rightarrow T$)

(A2) $I \oplus Y$ (or equivalently, $I \rightarrow \neg Y \wedge \neg I \rightarrow Y$)

Task 3. Formally argue that I holds, assuming F is true. [Hint: $H \vee \neg H$ is valid and analyse the two cases separately.]

Task 4. Construct a truth table to confirm this. You can automate this using the propositional formulas solver:

<http://logictools.org/index.html>

Simply enter your formula and inspect that the formula is only true for the rows that $I = 1$.

Alternatively, you can show this without using truth table. Simply prove that $F \wedge \neg I$ is unsatisfiable (i.e. all rows in the truth table are 0). This can be done by entering this formula in the formula box of the solver and make sure that you get “Clause set is false for all possible assignments to variables.” in the Result box. [Note: Set “using” to `dp11:old` (since `dp11:better` seems to have a bug).]

The murder mystery riddle

[30–35 mins]

There are three suspects for a murder: Adam, Brown, and Clark. It has been established beyond any reasonable doubt that exactly one of them is the killer. [So, two are innocents, one is guilty.] Your task is to figure out which one. You questioned Adam, Brown, and Clark one-by-one. Adam says “I didn’t do it. The victim was old acquaintance of Brown’s. But Clark hated him.” Brown states “I didn’t do it. I didn’t know the guy. Besides I was out of town all week.” Finally, Clark says “I didn’t do it. I saw both Adam and Brown downtown with the victim that day; one of them must have done it.” Assuming that the innocent men are telling the truth, but that the guilty man might not be, discover the killer.

Task 1. Carefully formulate the stated facts in propositional logic. Firstly, you must carefully pick your propositions. To get you started, here are some possible propositions:

- $I(A)$ — Adam is innocent.
- $H(C, V)$ — Clark hates the victim
- $T(B)$ — Brown was in town on the day of the murder
- $W(B, V)$ — Brown was with the victim on the day of the murder

Task 2. Clearly state some reasonable assumptions that arise from the semantics of English language. Here is an example:

(X) $W(B, V) \longrightarrow T(B)$, i.e., Brown being with the victim on the day of murder means that Brown was in town on the day of the murder.

Task 3. Make a conjecture who is the killer and formally argue that this is the case.

Task 4. Confirm this with the propositional formulas solver:

<http://logictools.org/index.html>

Set “using” to `dp11:old` (since `dp11:better` seems to have a bug).