FLOLAC 2015

Boolean Satisfiability and Synthesis Applications

Problem Set

Due on 2015/7/13 (Mon) 9:00 before lecture

1. **(Commutativity between Cofactor and Boolean Operations)** (20%)
   Given two Boolean functions f and g and a Boolean variable v, prove or disprove the following equalities:
   (a) $(\neg f)_v = \neg(f_v)$
   (b) $(f \ \wedge \ g)_v = (f_v) \wedge (g_v)$

2. **(Quantified Boolean Formula)** (40%)
   Given arbitrary Boolean functions f and g, prove or disprove the following equalities.
   (a) $\forall x.[f(x,y) \wedge g(x,z)] \leftrightarrow [\forall x.f(x,y)] \wedge [\forall x.g(x,z)]$
   (b) $\exists x.[f(x,y) \wedge g(x,z)] \leftrightarrow [\exists x.f(x,y)] \wedge [\exists x.g(x,z)]$
   (c) $\neg[\forall x.f(x,y)] \leftrightarrow \exists x.\neg f(x,y)$
   (d) $\forall x, \exists y.f(x,y) \leftrightarrow \exists y, \forall x.f(x,y)$

3. **(Quantifier Removal)** (20%)
   Given a quantified Boolean formula $\exists z.\varphi(x,y,z)$, suppose we would like to find some function $g(x,y)$ such that $\varphi(x,y,g(x,y))$ equals $\exists z.\varphi(x,y,z)$. What are the onset, offset, and don't-care set conditions for g in terms of φ?

4. **(ROBDD)** (20%)
   Let Boolean function $f = a'b + b'c + c'd$.
   (a) Draw the ROBDD of f with the variable order a, b, c, d (a on top).
   (b) Draw the ROBDD of f with the variable order b, d, a, c (b on top).