

Homework Assignment

[Compiled on July 7, 2015]

1. (40 Points) Apply the decision procedure for T_E to the following Σ_E -formulae. Provide a level of details as in slides #39 and #40.

(a) $f(x, y) = f(y, x) \wedge f(a, y) \neq f(y, a)$

(b) $f(g(x)) = g(f(x)) \wedge f(g(f(y))) = x \wedge f(y) = x \wedge g(f(x)) \neq x$

(c) $f(f(f(a))) = f(f(a)) \wedge f(f(f(f(a)))) = a \wedge f(a) \neq a$

(d) $p(x) \wedge f(f(x)) = x \wedge f(f(f(x))) = x \wedge \neg p(f(x))$

2. (20 Points) Apply the decision procedure for T_{cons} to the following T_{cons} -formulae. Please write down the call sequence to the *MERGE* procedure and draw the final DAG.

(a) $car(x) = y \wedge cdr(x) = z \wedge x \neq cons(y, z)$

(b) $\neg atom(x) \wedge car(x) = y \wedge cdr(x) = z \wedge x \neq cons(y, z)$

Hint: Apply preprocessing to the formulae if it is necessary.

3. (20 Points) Apply the decision procedure for quantifier-free T_A to the following Σ_A -formulae.

(a) $a\langle i \triangleleft e \rangle[j] = e \wedge i \neq j$

(b) $a\langle i \triangleleft e \rangle\langle j \triangleleft f \rangle[k] = g \wedge j \neq k \wedge i = j \wedge a[k] \neq g$

4. (20 Points) Consider a variant of the T_{cons} -satisfiability procedure in which Steps 2 and 3 are swapped. What is wrong with reversing these two steps? Identify a counterexample to its correctness: find a T_{cons} -unsatisfiable (conjunctive, quantifier-free) Σ_{cons} -formula that the incorrect procedure claims is satisfiable, and show the final DAGs for both the incorrect and the correct procedures.

Note:

Step 2: By the (left projection) and (right projection), ...

Step 3: For $i \in \{1, \dots, m\}$, *MERGE* $s_i t_i$