1.

(a) True.

Let $h(v, x_1, \ldots, x_n) = \neg f(v, x_1, \ldots, x_n)$.

$$
\begin{aligned}
(\neg f)_v &= h_v \\
&= h(1, x_1, \ldots, x_n) \\
&= \neg f(1, x_1, \ldots, x_n) \\
&= \neg[f(1, x_1, \ldots, x_n)] \\
&= \neg(f_v)
\end{aligned}
$$

(b) True.

Let $h(v, x_1, \ldots, x_n) = f(v, x_1, \ldots, x_n) \land g(v, x_1, \ldots, x_n)$.

$$
\begin{aligned}
(f \land g)_v &= h_v \\
&= h(1, x_1, \ldots, x_n) \\
&= f(1, x_1, \ldots, x_n) \land g(1, x_1, \ldots, x_n) \\
&= (f_v) \land (g_v)
\end{aligned}
$$

2.

(a) True.

$$
\begin{aligned}
\forall x.\ [f(x, y) \land g(x, z)] &= [f(0, y) \land g(0, z)] \land [f(1, y) \land g(1, z)] \\
&= [f(0, y) \land f(1, y)] \land [g(0, z) \land g(1, z)] \\
&= [\forall x.\ f(x, y)] \land [\forall x.\ g(x, z)]
\end{aligned}
$$

(b) False.

Counter example:

Let $f(0, y) = 0$, $f(1, y) = 1$, $g(0, z) = 1$, $g(1, z) = 0$.

$$
\begin{aligned}
\exists x.\ [f(x, y) \land g(x, z)] &= [f(0, y) \land g(0, z)] \lor [f(1, y) \land g(1, z)] \\
&= [0 \land 1] \lor [1 \land 0] \\
&= 0 \lor 0 \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
[\exists x.\ f(x, y)] \land [\exists x.\ g(x, z)] &= [f(0, y) \lor f(1, y)] \land [g(0, z) \lor g(1, z)] \\
&= [0 \lor 1] \land [1 \lor 0] \\
&= 1 \land 1 \\
&= 1
\end{aligned}
$$

(c) True.

Let $h(x, y) = \neg f(x, y)$

$$
\begin{aligned}
\neg[\forall x.\ f(x, y)] &= \neg[f(0, y) \land f(1, y)] \\
&= [\neg f(0, y)] \lor [\neg f(1, y)] \\
&= h(0, y) \lor h(1, y) \\
&= \exists x.\ h(x, y) \\
&= \exists x.\ \neg f(x, y)
\end{aligned}
$$

(d) False.

Counter example:

Let f(0, 0) = f(1, 1) = 1, f(0, 1) = f(1, 0) = 0.

$$\begin{aligned}\forall x, \exists y.\ f(x, y) &= [f(0, 0) \lor f(0, 1)] \land [f(1, 0) \lor f(1, 1)] \\ &= [1 \lor 0] \land [0 \lor 1] \\ &= 1 \land 1 \\ &= 1\end{aligned}$$

$$\begin{aligned}\exists y, \forall x.\ f(x, y) &= [f(0, 0) \land f(1, 0)] \lor [f(0, 1) \land f(1, 1)] \\ &= [1 \land 0] \lor [0 \land 1] \\ &= 0 \lor 0 \\ &= 0\end{aligned}$$

3.

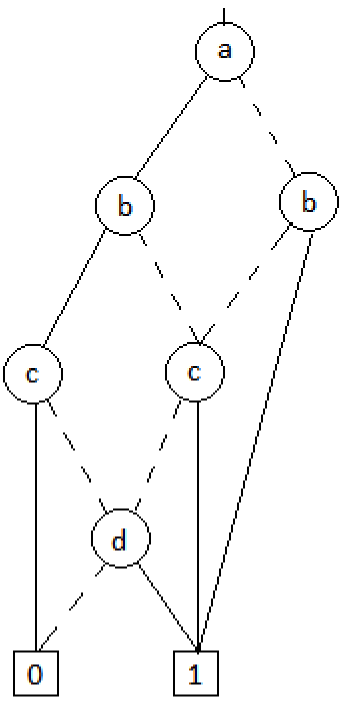
Onset: $\phi(x, y, 1) \land \neg\phi(x, y, 0)$

Offset: $\neg\phi(x, y, 1) \land \phi(x, y, 0)$

Don't-care set: $\phi(x, y, 1) \leftrightarrow \phi(x, y, 0)$

4.

(a)



(b)