# Logic: Homework 2

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Sixth Formosan Summer School on Logic, Language, and Computation, 2012

Please submit your solutions to me before 9:10AM on Monday, 3 September 2012.

1. Let $\Gamma : \textsc{List Prop}$ and $\varphi : \textsc{Prop}$. Prove that $\Gamma \models \varphi$ if and only if the list $\Gamma, \neg\varphi$ is not satisfiable.
2. How do you check satisfiability of a propositional formula and "finite" semantic consequence (i.e., a statement of the form $\Gamma \models \varphi$ where the number of propositional formulas in $\Gamma$ is finite) using the truth table method? Justify your answer.
3. Show that $\models \neg\neg\mathtt{A} \to \mathtt{A}$.
4. Show that $\models ((\mathtt{A} \to \mathtt{B}) \to \mathtt{A}) \to \mathtt{A}$.
5. Prove statement 1.4 in the proof of the soundness theorem.
6. Prove the weakening lemma.
7. Prove statement 3.2 in the proof of the reconstruction lemma.
8. Give the derivation of $\vdash_{\mathrm{NK}} \mathtt{A} \to \mathtt{A}$ constructed by the completeness theorem.
9. Let $\varphi := (\forall\, \mathtt{x}.\ \mathtt{P}\ \mathtt{x}) \to (\exists\, \mathtt{x}.\ \mathtt{P}\ \mathtt{x})$. Show that $\vdash_{\mathrm{NJ}} \varphi$ and $\models \varphi$.
10. Prove *Glivenko's Theorem*: $\Gamma \vdash_{\mathrm{NK}} \varphi$ if and only if $\neg\neg\Gamma \vdash_{\mathrm{NJ}} \neg\neg\varphi$ for every $\Gamma : \textsc{List Prop}$ and $\varphi : \textsc{Prop}$, where $\neg\neg\Gamma := [\, \neg\neg\varphi \mid \varphi \in \Gamma\,]$.