

Program Construction and Reasoning Exercises

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Guarded Command Language Basics

1. Swapping Booleans Verify:

```
[[ var  $a, b : \text{bool}$ ;  
    $\{a \leftrightarrow A \wedge b \leftrightarrow B\}$   
    $a := a \leftrightarrow b$ ;  
    $b := a \leftrightarrow b$ ;  
    $a := a \leftrightarrow b$ ;  
    $\{a \leftrightarrow B \wedge b \leftrightarrow A\}$   
]].
```

Hint: recall the definition $\text{true} \leftrightarrow a = a$, and that \leftrightarrow is associative: $(a \leftrightarrow b) \leftrightarrow c = a \leftrightarrow (b \leftrightarrow c)$.

2. Verify:

```
[[ var  $a, b : \text{bool}$ ;  
    $\{\text{true}\}$   
   if  $\neg a \vee b \rightarrow a := \neg a$   
    $\parallel a \vee \neg b \rightarrow b := \neg b$   
   fi  
    $\{a \vee b\}$   
]].
```

Loop and Loop Invariants

3. Prove the correctness of the following program:

```
[[ var  $x, y, N : \text{int} \{N \geq 0\}$ ;  
  
    $x, y := 0, 1$ ;  
   do  $x \neq N \rightarrow x, y := x + 1, y + y$  od  
    $\{y = 2^N\}$   
]].
```

4. Prove the correctness of the following program:

```
[[ var  $x, y, N : int \{N \geq 0\}$ ;  
  
    $x, y := 0, 0$ ;  
   do  $x \neq 0 \rightarrow x := x - 1$   
      $\parallel y \neq N \rightarrow x, y := N, y + 1$   
   od  
    $\{x = 0 \wedge y = N\}$   
]]
```

5. The following program non-deterministically computes x and y such that $x \times y = N$. Prove:

```
[[ var  $p, x, y, N : int; \{N \geq 1\}$   
    $p, x, y := N - 1, 1, 1$   
    $\{N = x \times y + p\}$   
   ; do  $p \neq 0 \rightarrow$   
     if  $p \bmod x = 0 \rightarrow y, p := y + 1, p - x$   
        $\parallel p \bmod y = 0 \rightarrow x, p := x + 1, p - y$   
     fi  
   od  
    $\{x \times y = N\}$   
]]
```