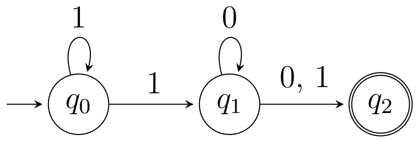
Suggested Solutions to Homework Assignment #2

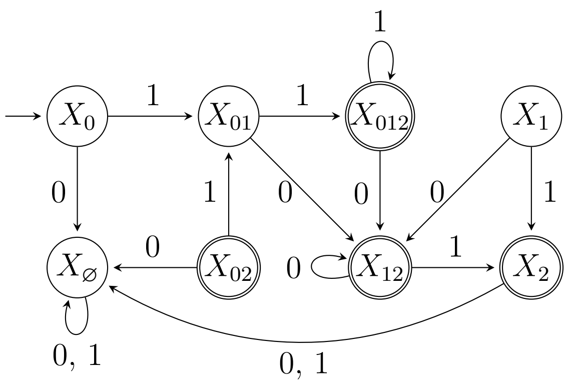
[Compiled on July 7, 2009]

1. (20 Points) Convert the following NFA N with $\Sigma = \{0, 1\}$ into a DFA D such that $L(D) = L(N)$.

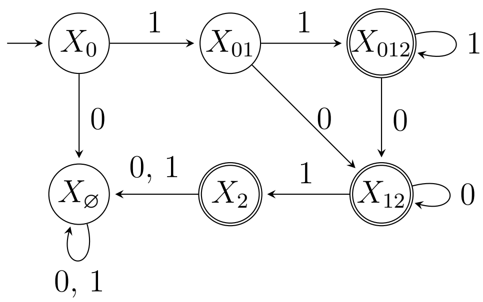


Solution.

After construction:

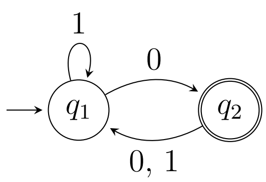


After removing unreachable states:



□

2. (20 Points) Convert the following DFA D with $\Sigma = \{0, 1\}$ into a regular expression R such that $L(R) = L(D)$.



Solution. $R = 1^*0((0+1)1^*0)^*$

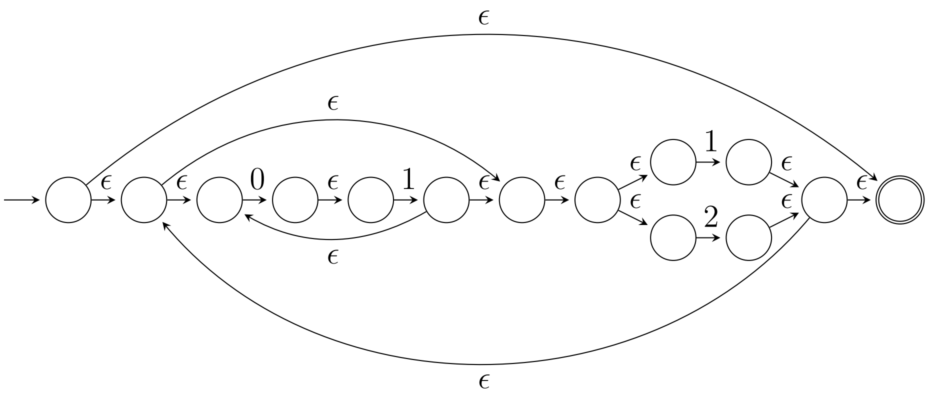
	$k = 0$	$k = 1$	$k = 2$
R_{11}^k	$1 + \epsilon$	1^*	
R_{12}^k	0	1^*0	$1^*0((0+1)1^*0)^*$
R_{21}^k	$0 + 1$	$(0+1)1^*$	
R_{22}^k	ϵ	$(0+1)1^*0 + \epsilon$	

□

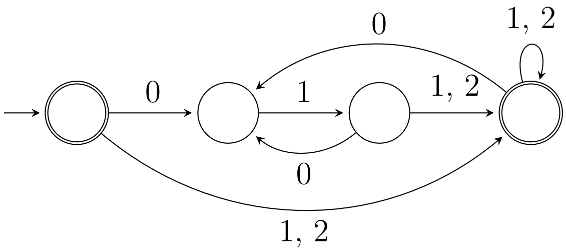
3. (20 Points) Consider the regular expression $R = ((01)^*(1+2))^*$ with $\Sigma = \{0,1,2\}$.

- Convert R into an equivalent NFA N with ϵ-transitions.
- Remove the ϵ-transitions of N.

Solution.



After removing all the ϵ−transitions:



□

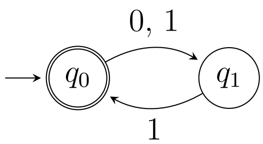
4. (20 Points) For each alphabet Σ and set A of input sequences, draw an NFA with Büchi acceptance that exactly accepts A.

(a) $\Sigma = \{0,1\}$ and $A = \{\alpha = a_1a_2\dots a_n\dots : \text{for all even } i, a_i = 1\}$

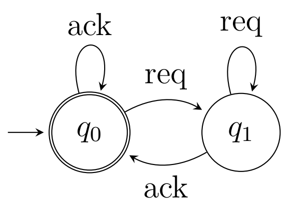
(b) $\Sigma = \{req, ack\}$ and $A = \{\alpha = a_1a_2\dots a_n\dots : \text{for all } i > 0, a_i = req$ implies that there exists $j > i$ such that $a_j = ack\}$

Solution.

(a)



(b)



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5. (20 Points) For each English sentence, write an equivalent monadic second logic formula.

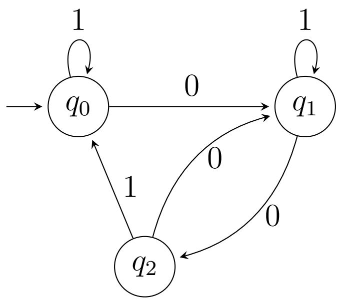
- The set P and the set Q have no common element.
- There is a maximal element in the set P. (Given two singleton sets x and y, x is smaller than y if $x < y$).

Solution.

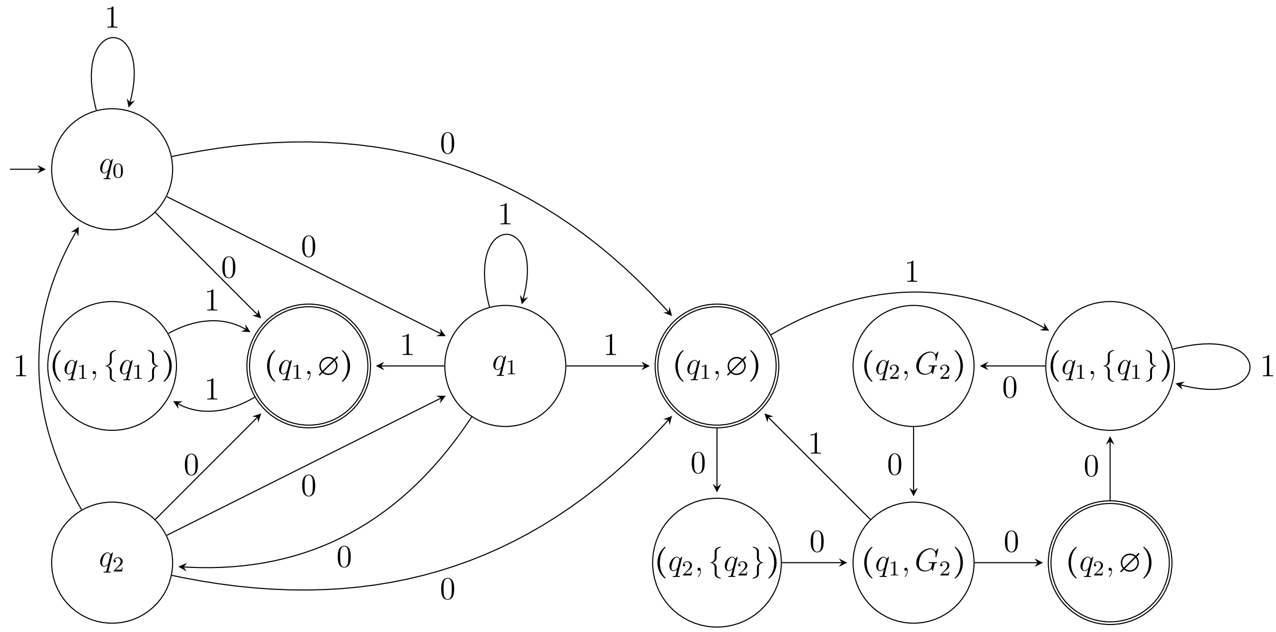
- $\forall x \in P, \forall y \in Q \neg(x = y)$
- $\exists x \in P, \forall y \in P(\neg(x = y) \rightarrow y < x)$

□

6. (0 Points) Convert the following NFA with Muller acceptance $\mathcal{F} = \{\{q_1\}, \{q_1, q_2\}\}$ into an equivalent NFA with Büchi acceptance.



Solution. In the following, we use G_2 to denote $\{q_1, q_2\}$.



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