

Homework Assignment #1

[Compiled on June 30, 2009]

Note

This assignment is due 9AM Thursday, July 2, 2009. Please write or type your answers on A4 (or similar size) paper. Put your completed homework by the due time on the lecturer's desk. **No late submission will be accepted. You may discuss the problems with others, but copying answers is strictly forbidden.**

Problems

- (20 Points) Prove the following tautological implication in sentential logic. P , Q , and R are sentence symbols.
 - $\{P \rightarrow (Q \wedge R)\} \models (P \rightarrow Q) \wedge (P \rightarrow R)$
 - $\{(P \wedge Q) \rightarrow R\} \models (P \rightarrow R) \vee (Q \rightarrow R)$
- (20 Points) Assume that a set Σ of wffs is finitely satisfiable and α is a wff. Show that either $\Sigma \cup \{\alpha\}$ or $\Sigma \cup \{\neg\alpha\}$ is finitely satisfiable.
- (20 Points) Consider the structure $\mathfrak{N} = (\mathbb{N}, S, 0, +, *)$. Write a first-order logic formula to define the set of odd numbers.
- (20 Points) Show that $(\mathbb{N}, +_{\mathbb{N}})$ and $(\mathbb{Z}, +_{\mathbb{Z}})$ are not elementarily equivalent by giving a sentence true in one but false in the other.
- (20 Points) Find a deduction (from \emptyset) for each of the following formulae.
 - $\exists x(\alpha \wedge \beta) \rightarrow \exists x\alpha \wedge \exists x\beta$
 - $Py \rightarrow \forall x(x \approx y \rightarrow Px)$