Logic
Homework for Lecture II

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Please answer as many of the following questions as you can, in Chinese or English, on the provided answer sheet and hand it to me on or before **July 9, 2008**. No delayed submissions will be accepted.

Do not feel pressured to complete *all* questions. The grading of your homework will not be based on how many questions you solve, but on how well you do compared with your classmates.

1 Natural Deduction for Propositional Logic

1. Give a derivation of $(P \wedge Q \to R) \to (P \to Q \to R)$.
2. Give a derivation of $(P \to Q \to R) \to (P \to Q) \to P \to R$.
3. Give a derivation of $P \wedge Q \to \neg(P \to \neg Q)$.
4. Give a derivation of $\neg P \vee Q \to (P \to Q)$.
5. Give a derivation of $\neg\neg\bot \to \bot$.
6. Show that the rule $(\bot\text{E})$ is *admissible* in system NK, i.e. show that from the premise $\bot$ you can derive any formula φ in NK.

   Thus, we can use $(\bot\text{E})$ as if it were a rule of NK.

2 Natural Deduction for First Order Logic

1. Can you give a derivation of $(\forall x.\varphi) \to (\exists x.\varphi)$ for any formula φ? Would you accept this inference step in a mathematical proof? Why or why not?
2. Show that $(\forall x.\varphi) \wedge (\forall x.\psi) \vdash_{\text{NJ}} \forall x.\varphi \wedge \psi$ for any formulas φ and ψ.
3. Show that $\vdash_{\text{NJ}} (\forall x.\varphi) \to \neg(\exists x.\neg\varphi)$.

3 Natural Deduction for Second Order Logic

1. Show that $\vdash_{\text{NJ}^2} \bot \leftrightarrow (\forall P.P)$.
2. Show that $\vdash_{\text{NJ}^2} \varphi \vee \psi \leftrightarrow (\forall P.(\varphi \to P) \to (\psi \to P) \to P)$.