2008 Formosan Summer School on Logic, Language, and Computation

Program Construction and Reasoning
Exercises for Day 3

Shin-Cheng Mu

July 8th, 2008

1. Derive the following program:

$$
\begin{array}{l}
|[\ \mathbf{con}\ N : \mathit{int}\{1 \leq N\}; f : \mathbf{array}\ [0 \cdot\cdot N)\ \mathbf{of}\ \mathit{int}; \\
\quad \mathbf{var}\ r : \mathit{int}; \\
\\
\quad \mathit{maxdiff}; \\
\quad \{r = (\uparrow p, q : 0 \leq p < q < n : f\ p - f\ q)\} \\
]|.
\end{array}
$$

   Hint: replace constant N by variable n, and use a loop that increments n near the end. You will need to strengthen the invariant and introduce an auxiliary variable.

2. The program *allzeros*, specified below, returns a length of the longest segment in the array f that contains only zeros:

$$
\begin{array}{l}
|[\ \mathbf{con}\ N : \mathit{int}\{0 \leq N\}; f : \mathbf{array}\ [0 \cdot\cdot N)\ \mathbf{of}\ \mathit{int}; \\
\quad \mathbf{var}\ r : \mathit{int}; \\
\\
\quad \mathit{allzeros}; \\
\quad \{r = (\uparrow p, q : 0 \leq p \leq q \leq n \wedge A\ p\ q : q - p)\} \\
]|,
\end{array}
$$

   where $A\ p\ q = (\forall i : p \leq i < q : f\ i = 0)$. Hint: similarly, replace N by n and use a loop that increments n near the end. You will need to introduce an auxiliary variable after strengthening the invariant. You will also find the following properties useful:

   (a) A holds for empty sequences. That is, $A\ n\ n$ is true for $0 \leq n \leq N$;

   (b) A is prefix-closed. That is, $A\ p\ q \Rightarrow (\forall i : p \leq i \leq q : A\ p\ i)$;

   (c) $x + (\uparrow p : \ldots : -p) = x - (\downarrow p : \ldots : p)$, where $\downarrow$ computes the minimum;

(d) and given predicate X, the proposition $s = (\downarrow i : 0 \leq i \leq n \wedge X\ i : i)$ (s is the smallest index between 0 and n that satisfies $X\ s$) equals the conjunction of the following propositions:

i. $0 \leq s \leq n$;

ii. $X\ s$;

iii. and $(\forall i : 0 \leq i < s : \neg X\ s)$.