Underivability (continued)

Lemma (instantiation). If $\Gamma \vdash \varphi$, then $\Gamma[\psi/v] \vdash \varphi[\psi/v]$.

Exercise. Define substitution $\varphi[\psi/v]$ and $\Gamma[\psi/v]$.

Corollary. $\nvdash_{\mathrm{NJ}} \neg\neg \mathtt{A} \to \mathtt{A}$.

ASSUME a derivation *d* of $\vdash \neg\neg \mathtt{A} \to \mathtt{A}$

GOAL contradiction

PROOF Reduce $\vdash \neg\neg \mathtt{A} \to \mathtt{A}$ to the law of excluded middle.

0 $\vdash \neg\neg(\mathtt{A} \vee \neg \mathtt{A}) \to (\mathtt{A} \vee \neg \mathtt{A})$

PROOF Apply the instantiation lemma to *d*, substituting $\mathtt{A} \vee \neg \mathtt{A}$ for $\mathtt{A}$.

1 $\vdash \neg\neg(\mathtt{A} \vee \neg \mathtt{A})$

2 $\vdash \mathtt{A} \vee \neg \mathtt{A}$ PROOF Combine 0 and 1 with the $\to$E rule.

3 QED. PROOF 2 should have been underivable.

Exercise. $\nvdash_{\mathrm{NJ}} (\neg \mathtt{B} \to \neg \mathtt{A}) \to (\mathtt{A} \to \mathtt{B})$.